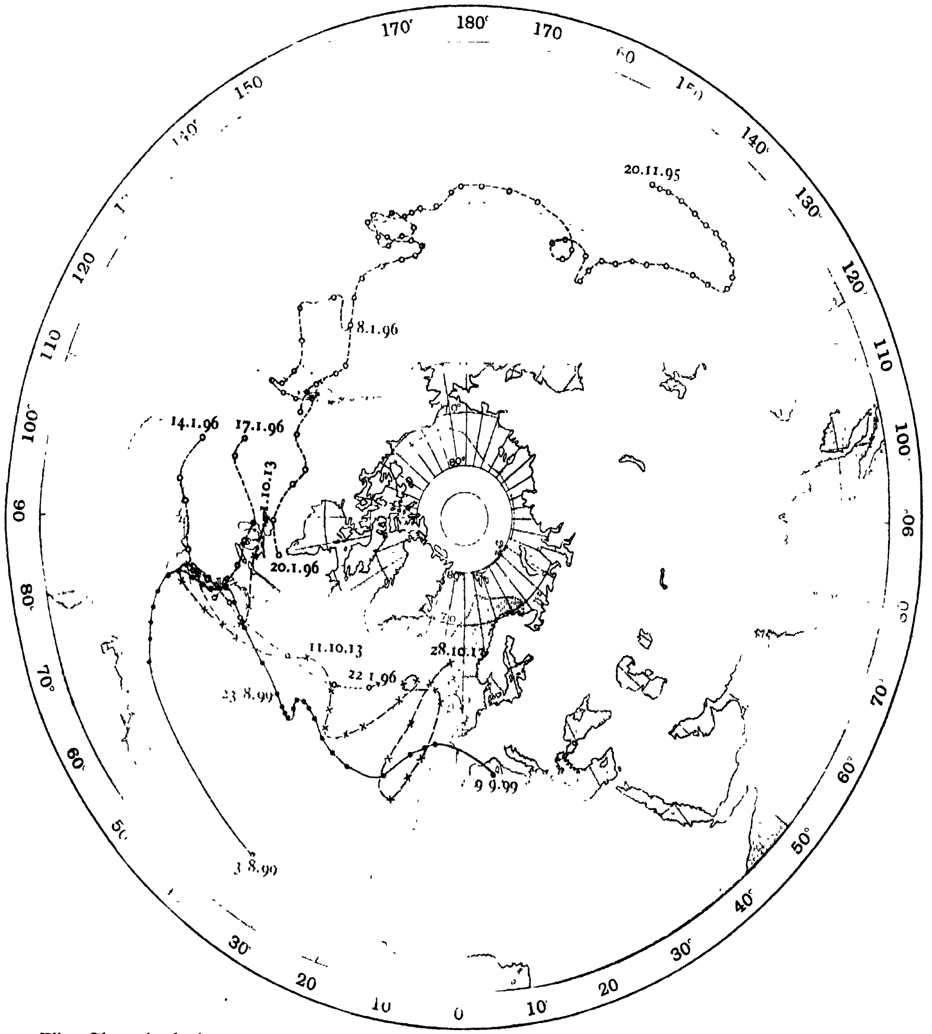
FRONTISPIECE

THE PATHS OF THE CENTRES OF SOME NOTABLE CYCLONIC DEPRESSIONS OF LONG DURATION. CHAPTER X.



The Chart includes

1. o---o---o The path of a *baguio* from lat. 15° N in the Western Pacific Ocean starting on 20.11.95. (A. McAdie.) Its position is shown day by day up to the American Continent with a duplication after 8.1.96. On arrival at the Rocky Mountains on 14.1 a redistribution takes place. A centre starts on that day from north of Mexico which is traced to the middle of the North Atlantic Ocean and lost on 22.1. Positions are shown, for morning and evening, on 15.1, 16.1, and 18.1.

A similar example is given by H. Harries, *Q. J. Roy. Met Soc.*, vol. XII, p. 10, 1886.

2. •—•—• The path of a cyclonic depression from Mid-Atlantic in 12° N on 3.8.99 which reached the West Indies in five days and passed on to the Mediterranean. (M.O.) Daily stages are marked from 8.8 to 9.9. There is an interval of five days between 3.8.99 and the next point.

3. x--x--x The irregular path of a cyclonic depression from the Great Lakes to the Faroe. (McAdie.) Daily stages are marked from 1.10.13. On M.O maps there is a duplication of centres on 26.10.

Manual of Meteorology

PART IV

THE RELATION OF THE WIND TO THE DISTRIBUTION OF BAROMETRIC PRESSURE

BY

SIR NAPIER SHAW, Sc.D., F.R.S.

DIRECTOR OF THE METEOROLOGICAL OFFICE, READER IN METEOROLOGY IN THE UNIVERSITY OF LONDON; HONORARY FELLOW OF EMMANUEL COLLEGE, HONORARY DOCTOR OF LAWS OF THE UNIVERSITY OF ABERDEEN; HONORARY DOCTOR OF SCIENCE OF THE UNIVERSITIES OF DUBLIN AND MANCHESTER AND OF HARVARD UNIVERSITY, CAMBRIDGE, MASS.; HONORARY FOREIGN MEMBER OF THE AMERICAN ACADEMY OF ARTS AND SCIENCES, BOSTON, AND OF THE REALE ACCADEMIA DEI LINCEI, ROME

CAMBRIDGE
AT THE UNIVERSITY PRESS
1919

CAMBRIDGE UNIVERSITY PRESS

C. F. CLAY, MANAGER

LONDON : FETTER LANE, E.C. 4

LONDON : H. K. LEWIS AND CO., LTD.,
136 Gower Street, W.C. 1

LONDON : WILLIAM WESLEY AND SON,
28 Essex Street, Strand, W.C. 2

NEW YORK : G. P. PUTNAM'S SONS

BOMBAY
CALCUTTA } MACMILLAN AND CO., LTD.
MADRAS

TORONTO : J. M. DENT AND SONS, LTD.

TOKYO : MARUZEN-KABUSHIKI-KAISHA

PREFACE

WITHIN the past four years urgent questions have been addressed to the Meteorological Office from many quarters about the winds. Some of them refer to the winds at the surface, a subject of immemorial antiquity which has only within the last score of years been subjected to comparatively accurate measurement; and others refer to the winds of the free atmosphere beyond the reach of the highest point upon which an anemometer could be fixed.

The work of the Meteorological Office aided by the contributions of data from various departments of the naval, military and air-services has provided the material for answers to these questions, but the first conclusion derived from the study of the material is that the answers to all the questions cannot be treated separately. They all form part of a description of the structure of the atmosphere, a structure so complicated, even so perverse, as to make the attempt to describe it intelligibly without some guiding principle, or to deal with it piecemeal, hopeless.

We have found a guiding principle of great practical utility in the relation of the wind to the distribution of pressure which can be deduced from the assumption that as a general rule the motion of air in the free atmosphere follows very closely the laws of motion under balanced forces depending on the spin of the earth and the spin in a "small circle" on the earth. And therefore, in order to provide the best available answers to the questions put to us we have studied the relation of the winds to the distribution of pressure at the surface and in the free atmosphere.

This study has led to setting out what amounts almost to a general meteorological theory. It forms Part IV of this manual and

includes within its scope the best answer which we are able to give to the general questions put to us.

Behind it lies the vast accumulation of facts obtained by the industry and perseverance of meteorological observers all over the world which are represented compendiously for practical meteorologists by a series of normal values of meteorological elements of every kind. These are summarised in Part I as a general survey of the globe and its atmosphere which is based upon the great but unfortunately still incomplete work of Hildebrandsson and Teisserenc de Bort, *Les Bases de la Météorologie dynamique*.

But anyone who takes an intelligent interest in the structure of the atmosphere, whether he regards it in detail or thinks only of its more general features, must have a working knowledge of the physical properties of air, and that is no slight matter. Maxwell's wonderful *Text-book of Heat* with all its digressive chapters might have been written for the purpose, for all that it contains is extraordinarily appropriate. Only one additional chapter, on the difficult subject of the thermodynamical properties of moist air, is required.

The physical properties of air form the subject of Part II.

Part III contains the formal setting out of the dynamical and thermal principles upon which theoretical meteorology depends, and which find their application in Part IV. It is necessarily technical but again its main outlines are sketched by the hand of a perfect master of the art in Maxwell's *Matter and Motion*.

The whole is preceded by a historical introduction and a statement of the position of the general meteorological problem at the present day; because the history of the study of weather forms a striking example of the interaction of the progress of science and the creation of the instruments which it uses.

Part IV is issued in advance because what is contained therein has not hitherto been presented in a collected form. It represents the progress made chiefly by those who have been associated in the work of the Meteorological Office in the past twenty years. We owe our success to the fortunate circumstances of our meteorological *terrain*.

For the other parts of the subject all meteorologists have the same sources of reference open to them if they care to use them. Our concern in this work is to present a summary of them in the most handy form for conveying an idea of the information which is available. For the survey of the meteorology of the globe Bartholomew's *Atlas of Meteorology* by Buchan and Herbertson is in itself an admirable compendium.

The special climatological atlases, of Russia by General Rykatchef, of India by Sir John Eliot, of Canada by Sir Frederick Stupart, the great work on the climatology of the United States of America by A. J. Henry and the less complete but still notable work by C. M. Delgado de Carvalho on the Meteorology of the United States of Brazil invite contributions to the common stock of knowledge on the part of other countries so that the student of meteorology may not continue to be dependent upon the data in their original form, which are only contained in a few of the libraries of any country.

The physical and dynamical principles upon which the processes of weather depend are the common property of all students of physics. If those to whose care the progress of physics is entrusted had taken the physical problems of the atmosphere under their charge as their predecessors did before the advent of the electrical era one half at least of this book might have been more effectively dealt with by other hands.

A work of this kind necessarily depends in very large measure upon illustrations which often represent, in the most succinct manner, results of observations which cannot be transcribed in words or formulae. The original drawings are the only satisfactory evidence for writer or reader because in the gradual development of the science what at one stage of our knowledge appears to be a superfluous accident may become the starting point of a new advance. The author and the Meteorological Committee, at whose instance this work was undertaken, desire here to place on record their acknowledgment to the Controller of H.M. Stationery Office, the Board of Trade, the Ordnance Survey and the Advisory Com-

mittee for Aeronautics for permission to use illustrations which have appeared in the publications of H.M. Government.

And similar acknowledgment is due to the Royal Society, the Royal Meteorological Society, the Carnegie Institution of Washington, Professor McAdie of Harvard University and particularly to Captain C. J. P. Cave of Ditcham Park.

Particulars of the extent to which illustrations have been borrowed are set out in the list contained in pp. xiv to xvi.

The author desires also to express his thanks to an old friend and colleague, Mr J. B. Peace, Printer to the University of Cambridge, for the care which he has given, in the difficult circumstances of the later stages of a great war, to the arrangement of the book and the form of the illustrations.

NAPIER SHAW

METEOROLOGICAL OFFICE, LONDON
9 *December* 1918

A preliminary issue of a limited number of copies of this part of the work for official use has enabled me to obtain from colleagues and friends some corrections and suggestions which have now been incorporated in the text or added as notes.

N. S.

17 *May* 1919

TABLE OF CONTENTS

PART IV

LIST OF ILLUSTRATIONS

NOTE. Many of the diagrams are copied, with modifications in some instances but more generally in the original form, from the publications to which reference is made. Those marked * are from Official Publications of H.M. Government and permission for their reproduction has been obtained from the Controller of H.M. Stationery Office. The mark † refers to the *Reports of the Advisory Committee for Aeronautics*.

CHAPTER II

CHAPTER IV

CHAPTER V

CHAPTER VI

CHAPTER VII

CHAPTER VIII

CHAPTER IX

Chapter X

Chapter XI

PLATES

PHOTOGRAPHS OF CLOUD-SHEETS FROM AEROPLANES

PART IV

THE RELATION OF WIND TO THE DISTRIBUTION OF BAROMETRIC PRESSURE

CHAPTER I

The details of the determination of the pressure-gradient and the wind

WE have seen in the description of the general circulation of the atmosphere (Part I, chap. V) that the wind as observed in the usual way near the surface is generally related to the distribution of pressure in accordance with Buys Ballot's law. In the northern hemisphere pressure is lower on the left of the flow of the wind and higher on the right. In the southern hemisphere the reverse is the case; and in either hemisphere the closer the isobars are together the greater, as a rule, is the velocity of the wind.

The relation which these words express is rather vague. At well-exposed stations on our coasts the inclination of the observed wind to the run of the isobars is generally about 30° towards the side of lower pressure but it varies between wide limits; sometimes the wind follows even the local deviations of the line of the isobars with surprising fidelity as in the map for 8 a.m. of April 9, 1908, but occasionally the wind may cross the isobar at 45° or more. At inland stations, particularly those in hilly or mountainous districts, the law has many exceptions. The relation of the velocity to the gradient of pressure also shows variations within wide limits so that if the surface-wind be taken as the basis of computation no satisfactory expression can be given for a relation between wind and gradient as representing the underlying principle upon which Buys Ballot's law depends.

But in Part III we have seen that under certain conditions there ought to be a relation between the flow of air and the distribution of pressure in the atmosphere of a rotating globe such as, without serious error, we may consider our earth to be. The relation which is there established is that expressed by the equation

$$s = 2\omega v\rho \sin\phi \pm v^2\rho \cot r/E, \qquad \ldots\ldots(\gamma)$$

where s is the pressure-gradient, or tangent of the slope of the isobaric surface,

v the velocity of the wind,

ρ the density of the air,

ϕ the latitude of the place of observation,

r the angular radius of the small circle which marks the path of the air at the moment,

E the radius of the earth,

ω is the angular velocity of the earth's rotation.

The double sign $\pm$ means that the formula is different according as the path of the air deviates from a great circle to the left, or to the right. If it turns to the left the small circle which identifies the curvature of the path for the moment has low pressure at its centre. It therefore belongs to a cyclonic system, and the $+$ sign is to be used: but if the air deviates to the right the small circle in which it is moving has higher pressure at its centre; the circulation is anticyclonic and the $-$ sign is to be used.

The conditions under which this equation holds are that the motion of the air is along a horizontal surface, that the resultant of the forces acting upon the air is perpendicular to the direction of motion so that there is no immediate acceleration or retardation of the speed and the resultant force must be of the proper magnitude to balance the kinematic effect of the spin of the earth and the spin of the air in the small circle which marks its path.

These conditions clearly cannot be satisfied in the case of the air that flows along the surface because there is always the friction of the surface which absorbs momentum from the flowing stream and consequently appears as a retarding force in the line of motion. It is by the sacrifice of its momentum that wind produces the effects upon land and water with which we are familiar. The energy of sea-waves comes from the motion of the wind, as does also the energy that is displayed in the destructive effects of a gale, a hurricane or a tornado. Consequently, in so far as Buys Ballot's law is a manifestation of the relation of the wind to the distribution of pressure indicated by the equation for gradient wind, it is under the most unfavourable conditions that the manifestation takes place at the surface. The fact that a relation can be recognised so frequently in conditions so unfavourable certainly implies that the relation of wind to pressure in some form or other is an important principle in the structure of the atmosphere.

William Ferrel[1] had already used the effect of the earth's spin as the basis of a scheme for the general circulation of the atmosphere. Guldberg and Mohn[2] attributed the difference between the computed results and those observed for the surface-wind to friction and a coefficient of friction was evaluated; but when the soundings of the upper air made with kites by W. H. Dines from 1903 onwards were referred to our maps the fact of the nearer approximation to the computed value both as regards direction and velocity shown by the winds of the upper air at once challenged attention.

The question was examined by E. Gold in a Report on Barometric Gradient and Wind-force[3]. The practical conclusion of the examination was that if the wind at the 500 metres level was compared with the surface-gradient there was on the average close agreement, the deviation being sometimes one

[1] *A popular treatise on the winds.* New York, 1889.

[2] 'Studies of the movements of the Atmosphere' (original paper revised by authors and translated by Cleveland Abbé), *Smithsonian Miscellaneous Collections*, vol. 51, No. 4, XI, p. 122.

[3] M. O. Publication, No. 190, 1908.

way and sometimes another. And Gold added in the same report a theoretical proposition easily verified by common observation and of great importance. He showed that if the negative sign be taken in the gradient equation and the equation be solved as a quadratic, for the determination of the wind-velocity v corresponding with a given gradient s, the roots become imaginary if the curvature is above a certain limit. From which it follows that if the balance of wind and gradient is the principle upon which the structure of the atmosphere is based the winds near the centre of an anticyclone, where the curvature of the isobar must be relatively great, cannot exceed a certain small limiting value, whereas no such limitation is applicable in the case of a cyclonic depression. This gave a dynamical explanation of the well-known fact that the central region of an anticyclone is always an area of very light winds whereas there is practically no limit to the velocity of the winds in the small circles near the core of a tropical revolving storm or a tornado.

Up to that time it had been customary to consider all such dynamical problems from the point of view of the motion of the air resulting from the effect of a finite difference of pressure, and Gold gave a calculation of the time which would be required for the velocity to adjust itself to the pressure-gradient, but if we consider that the process of adjustment is constantly going on and that in most unfavourable circumstances Buys Ballot's law gives evidence of the adjustment, we are left to surmise what uncompensated differences of pressure are likely to be found in the free atmosphere where there are no disturbing forces such as are found at the surface and where an infinitesimal change in the distribution of pressure or in velocity at once sets up the process of readjustment. We are precluded from supposing that the retarding force of "friction" at the surface extends to great heights in the atmosphere by the direct action of viscosity because Helmholtz has shown that any such effect is negligible[1].

The investigation of the *Life-History of Surface Air-currents*[2] showed that in actual practice air must be regarded as travelling over long tracks of sea and land with very little change of velocity from hour to hour or even, on occasions, from day to day and suggested that such incidents as being caught in the ascending current of a rain-storm or shower, the most likely disturbance of an even progress, must form a very small part of the life-history of air-currents; that the greater part must be made up of such steady motion as we see in the case of clouds in common weather which travel for long distances with very little change of speed.

If we consider other cases in which long continuous journeys are made such as that of a ship pursuing a similarly even course from one port to another three thousand miles away we are not, as a rule, much concerned with the acceleration of the start or the retardation of the landing but with the circumstances of the motion of the long voyage so nearly uniform that we may regard the ship as moving under balanced forces. So with the winds in meteorology, it is not that portion of their history in which they are starting or stopping, accelerated from quiescence or brought to rest by some sudden difference of

[1] *Sitzungsber. K. P. Akad. Wiss.*, 1888, p. 647. [2] M. O. Publication, No. 174, 1906.

pressure operating for a short time, that we have to think about so much as the longer period when they keep on their uniform way under balanced forces like a train or a motor-car that has got its speed and is making a run on a long stretch. There will, of course, be variations in the gradient of the road that will alter the rate of motion, sometimes slowing it down, sometimes speeding it up, but the chief feature of the motion is the balance between the motive forces of the engine and the resisting forces of the air and the wheels.

If we may be permitted to consider the air as having in like manner arrived at an established state of motion, without being obliged to trace the original causes of its motion or speculate upon its arrest, we may limit our consideration to the progress of its motion; in ordinary circumstances this will show very little variation of speed in the course of hours in comparison with the possible effects of the force of pressure which has been operative during those hours, in a direction transverse to the air's motion; so that the function of pressure-distribution seems rather to be to steer the air than to speed it or stop it. So it will be more profitable to consider the "strophic" balance between the flow of air and the distribution of pressure as an axiom or principle of atmospheric motion which, at the moment, cannot be either directly verified or directly contradicted, and examine the phenomena from that point of view. This principle was enunciated in a paper before the Royal Society of Edinburgh[1] in 1913 as follows: *In the upper layers of the atmosphere the steady horizontal motion of the air at any level is along the horizontal section of the isobaric surface at that level and the velocity is inversely proportional to the separation of the isobaric lines in the level of the section.*

In this statement the effect of the curvature of the path of the moving air was disregarded and the cases under consideration were therefore limited to those in which the air is moving approximately along a great circle and this limitation requires notice. There are two terms on the right-hand side of the equation for gradient-wind so that for a given value of the velocity v two causes contribute to the making of the balance of the gradient s. One part which we may write s_ω is represented by the term $2\omega v\rho \sin\phi$ and is therefore due to the earth's rotation, we may call that part the "geostrophic" component, the other part which we may denote by s_r is due to the spin in a small circle of angular radius r; this we may call the "cyclostrophic" component. It is evident that as the latitude ϕ becomes less, that is as we move nearer to the equator, the geostrophic component becomes relatively of less importance because the factor $\sin\phi$ diminishes and at the equator, where $\phi = 0$, the effect of the rotation of the earth has vanished, the only possible balance of wind and pressure is the cyclostrophic balance such as we may appreciate in rapidly rotating whirls of fluid. On the other hand if the motion is in greater and greater "small circles" the relative importance of the cyclostrophic component becomes less and less until when the motion is along a great circle $\cot r$ becomes zero and the rotation of the earth alone is operative to maintain the balance between velocity and pressure. The wind computed according to the complete

[1] *Proc. R.S.E.*, vol. XXXIV, p. 78, 1913.

formula we call the "gradient-wind" and if the curvature of the path, rightly or wrongly, be disregarded and the velocity computed as balancing the pressure with the aid of the earth's spin alone we call the computed value the "geostrophic wind."

To assume that this balance of wind and pressure in the upper air is an operative principle of atmospheric structure may be thought a hazardous mode of procedure and it requires the most scrupulous examination, but the proper course seems to be to accept it at least until the proved exceptions are numerous enough to show that, under the prescribed conditions of motion approximately in a great circle, finite differences of pressure do exist in the air without the compensating velocity in the air-currents. It need not be supposed that the balance is always strictly perfect but only that in ordinary circumstances the accelerating forces operating in the air are so small in relation to the pressures that we measure, that they are beyond our powers of observation. In the remainder of this chapter we propose to examine the question of the direct comparison of the actual wind with the wind computed from the gradient.

First it must be remembered that the determination either of the gradient or of the actual wind at any level is attended with great practical difficulty. The gradient is determined by plotting observations of pressure on a map and drawing isobars across the area. The pressures are observed by different observers and accuracy in the collection of simultaneous values depends in the first place upon punctuality, too soon being as bad as too late, and upon the careful attention to the organisation of the network of stations. The observed pressures have to be reduced to sea level by a process which may introduce quite appreciable errors if the observing station is some hundreds of feet up. The stations of the Meteorological Office are about fifty miles apart over the British Isles and the seas have to be bridged by connexion with stations on either side or by wireless reports from distant ships. With stations so wide apart isobaric lines can be drawn with reasonable certainty so far as their general run is concerned but there may be details of variation which the observations do not show. A local gradient, for example, for a width of say five miles along a coast may correspond truly with a local wind; the local wind would be obtained by direct observations, the local gradient could only be obtained by the most elaborate process because the pressure-difference over five miles necessary to balance a geostrophic wind of gale force is not more than half a millibar. Slight variations shown on a barogram by fluctuations in the record which we call embroidery cannot be represented on the map without elaborate investigation quite beyond the scope of the daily service which provides the maps from which gradients are taken.

Gradients for higher strata than sea level are practically unattainable at present except by calculation, and for adequate calculation accurate information of the temperature at different levels above the places of observation is required but is not yet available.

But if there is difficulty about obtaining the gradient the difficulty of

obtaining a measure of the wind to compare with it is still greater. If the map is drawn for sea-level winds at sea-level are wanted. The nearest approach to those are winds at the surface and these are subject to interference of so many kinds that it is difficult even to say what a measure of wind exactly means.

The reader may be surprised to learn that measuring the wind is really a most difficult operation, but he may realise the truth of the statement when he understands that twenty years ago, on account of the inherent difficulty of the subject, the Meteorological Office had to be content to publish values of wind-velocity which were known to be in error by twenty-five per cent., and even now the quotation, without reference, of a reading of an anemometer from a considerable number of meteorological publications is no guarantee that an error of that order of magnitude is not involved. By the turn of the century much had been done and the accuracy of wind measurements has been still further improved in recent years by the use of wind channels set up for aeronautical investigations. In working conditions an ordinary measure of wind as represented on a weather-map of the British Isles and the neighbouring parts of the continent and the islands of the Atlantic is obtained for the most part by estimation according to the Beaufort scale, and the same is true universally of the measures of wind at sea which are generally available for meteorological study. In order to express these estimates in terms of velocity the scale of equivalents of the numbers of the Beaufort scale given in chap. VI of the Introduction is employed. The scale was obtained[1] by taking the mean value of many individual observations which show very considerable diversity among themselves and, strictly speaking, it can only be regarded as applicable when in like manner the means of a large number of estimates are under consideration. Any individual estimate of wind-force is liable to the various causes, partly peculiar to the locality or the special conditions of weather and partly personal to the observer, which account for the large range of estimates included under the same number of the scale in the formation of the table of equivalents. At a few observatories measures of the wind for use in the preparation of weather-maps are obtained by anemometers and the figures are transformed into the numbers of the Beaufort scale for transmission by telegraph by the use of the same table of equivalents. In such cases there are no uncertainties in the measurement such as are inseparable from personal estimates but the peculiarities of the exposure of the anemometers are at present even more disturbing than the vagueness of personal estimates. The older observatories were provided with recording cup-anemometers of the Robinson type and these are very heavy instruments which have to be supported on substantial structures and are generally installed on large buildings which, with their surroundings, affect the flow of air past the instrument with eddies of various kinds. The subject is at present not at all fully investigated and it has only recently been pointed out[2] in a summary of the occurrence of gales in different localities that, for this reason, the observatories of the Meteorological

[1] *The Beaufort Scale of Wind-Force*, M. O. Publication, No. 180, 1906.

[2] *The Weather of the British Coasts*, M. O. Publication, No. 230, p. 34, 1918.

Office are characterised by a singular and anomalous freedom from gales which is really attributable to the exposure of the anemometers. No doubt the position of the question can be improved by a discussion of the available data when a standard of reference can be agreed upon. At present the only standard that seems in any way adequate is the geostrophic wind of straight isobars and that is acceptable only if we may assume that the geostrophic wind for straight isobars is a real equivalent of the actual flow of air undisturbed by surface effects and local eddies.

With the newer tube-anemometers of the Dines type the position is considerably improved. The vane is generally exposed on a slender mast forty feet above the ground and can be set up in an open situation with only a small hut at its base, so that the whole structure offers very little cause for interference with the wind. In that case it is only the ground and the irregularities of the relief of the region which produce disturbing eddies; but even these effects are at present unknown in detail, nor is it possible to suppose that any simple relation can exist which will make the records of anemometers at different places strictly comparable and afford a satisfactory standard of reference. Allowance must be made for differences in height and the variation of wind with height is not only very large but it is in itself an extraordinarily complicated question. A great deal of light has recently been thrown upon it by G. I. Taylor's investigations of eddy-motion in the atmosphere. The theory[1] confirms what has all along been dimly foreshadowed by observation, that the variation of wind with height depends upon the locality and the nature of the surroundings: it is different for sea and for land, for a town and for the open country, and so on: it also depends upon the undisturbed velocity of the wind in the upper air, upon the surface temperature of the ground, and hence upon the time of day and the season of the year. For the purpose of his investigations Taylor relies upon the geostrophic wind as representing the undisturbed wind in the upper air and no other standard of reference seems possible. Here we may also note that in certain circumstances the wind at or near the surface may show virtual independence of the general distribution of pressure in consequence of the direct effect of the gravitation of cold air down the slopes of hills at night, a form of wind which in Part I we have called *katabatic*, and equally in the daytime air-currents near the surface may represent the levitation due to the warming of slopes in the sun which gives rise to winds which we have called *anabatic*, and which are illustrated by the familiar phenomena of land and sea breezes.

Hence it will be seen that a measure of wind by estimate or by anemometer near the surface is subject to so many local influences and other possibilities of discrepancy that it is not practicable to build upon it any reasonable picture of the structure of the atmosphere. It offers us a number of problems which we may seek to explain by tracing the effect of local causes upon the upper wind that really belongs to the structure of the atmosphere.

[1] 'Phenomena connected with Turbulence in the Lower Atmosphere,' *Proc. Roy. Soc.* **A**, vol. XCIV, p. 137, 1918.

In recent years direct observations of winds in the upper air by pilot-balloons or other similar means have given us measurements which certainly belong to the structure of the atmosphere and enable us to approach the question of the relation of wind-velocity to pressure-distribution from a new starting point. We might take the wind at 500 metres or some other height as a standard of reference and compare it with the gradient-wind, but the relation which has been assumed, and which we seek to justify, refers to wind and gradient at the same level and we have no precise measure of the gradient at the level of 500 metres; and if we had, we have still to decide whether the influence of the surface ceases at that level. Hence, we come back to the method of assuming the correspondence between actual wind and geostrophic wind in the free air for straight isobars and tracing all the consequences that can be inferred therefrom. In pursuance of this plan we shall put together in the next chapter the facts of the relation of the surface-winds to the distribution of pressure at sea level so far as we know them. We may pass on then in the following chapters to consider the facts about the variation of wind with height and subsequently see how far these facts can be explained upon the hypothesis which we have set out as the first law of atmospheric motion.

CHAPTER II

The relation between the surface-wind and the geostrophic wind at sea level. Geostrophic wind-roses

UNTIL the question is examined in its various aspects, some of which have been referred to in the preceding chapter, the relation between the surface-wind and the gradient-wind for sea level seems a simple question. It used to be treated simply as a question of determining the relation between wind-force and the gradient or the separation of the isobars, which, for the geostrophic wind, is inversely proportional to the wind-velocity. It was on those lines that values were obtained in 1882 for the average relation of the wind at Kew Observatory to the barometric gradient[1]. Subsequently the relation of the surface-wind to the geostrophic wind as watched from day to day in the study of weather-maps was formed into a sort of working rule that the current of air of the upper regions lost one-third of its velocity over the sea, and two-thirds over the land; so that, for the same distribution of pressure, the wind recorded in open inland country would have one-half of the velocity appropriate to the same distribution at sea, a conclusion which was exemplified by an occasion when parallel isobars from the West-South-West covered the whole country and gave force 8 at the exposed stations on the Western coasts and force 5 at the stations inland and on the Eastern coasts. Naturally the coast stations if they are on a well-exposed flat shore belong to the régime of the sea, for off-sea winds, and to the inland for off-shore winds.

As a rough working rule this is still a useful form of note to carry in the memory, but in the critical examination of the question of the relation of the observed wind to the geostrophic wind at sea level no such simple generalisation can be allowed.

The mere consideration of the diurnal variation of wind-velocity as shown by the anemometers at the observatories is sufficient to make it clear that no single number can express the relation of the observed wind to the geostrophic wind for any locality. A marked diurnal variation of wind-velocity is shown in all the hourly normals of wind-velocity even in winter and still more in summer. There is, on the other hand, no observational evidence for the existence of a diurnal variation of barometric gradient; and we therefore assume that for the average of a large number of observations the surface-winds of the day-hours and night-hours may be referred to the same geostrophic wind[2]. The

[1] G. M. Whipple, *Q. J. Roy. Met. Soc.*, vol. VIII, p. 198, 1882.

[2] The differences of corresponding hourly values between Richmond (K.O.) and Aberdeen or between Richmond (K.O.) and Cahirciveen show that there is no appreciable diurnal variation in the general gradient for the westerly component or the southerly component of the winds, but it should not be forgotten that there may be a local gradient on crossing a coast line due to the difference in the régime of temperature distribution in the vertical over

hourly values of wind-velocity are available in tabular form for the four observatories, Aberdeen, Cahirciveen (Valencia Observatory), Falmouth, and Richmond (Kew Observatory), since 1868 and for Eskdalemuir since 1911. The average factor of relationship between the observed wind and the geostrophic wind at different times of the day in different seasons of the year will vary to the same extent as the average values of the observed wind-velocities as set out in the following table.

TABLE I.

Observatory	January				July			
	Minimum wind	Hour	Maximum wind	Hour	Minimum wind	Hour	Maximum wind	Hour
	m/s		m/s		m/s		m/s	
Aberdeen	4·4	4	4·9	13–14	2·4	3–5	4·3	13–14
Cahirciveen	6·3	4–6	7·2	14	3·6	5	5·8	14–15
Falmouth	4·9	6–8	6·0	13	2·9	4–5	5·0	14
Richmond (K. O.)	3·3	1–4	4·3	12–14	1·8	3–5	4·2	14
Eskdalemuir	5·1	7	6·5	12	2·7	4	5·6	14
Eiffel Tower (304 m.)	8·8	14	10·8	23	5·4	9	9·0	24

It should be noted that these figures are averages: there are days when the diurnal variation of the wind is obliterated by conditions of weather and there must also be days in which it is much more marked than the average, and in consequence the factor of relation of surface-wind to geostrophic wind will certainly have a wider range than that shown in the table. When the averages are expressed as fractions of the geostrophic wind there will obviously be large variations depending upon the time of day and the season of the year for which comparisons are made. For the British observatories the lowest values will be obtained for the night or the early hours of the morning and the highest with almost unanimous concurrence at 14 h, two hours after Greenwich noon. For the Eiffel Tower, for which corresponding values are also included in the table, the opposite is the case.

The diurnal variation of the velocity of the wind was explained by Espy[1] and Koppen[2] as due to the effect of the temperature of the surface upon the

the land as compared with that over the sea for which no measurements are available. An example of a strong local wind attributable to the coastal gradient is cited in *Barometric gradient and wind-force*, M. O. 190, p. 9. E. L. Hawke has also shown that there is a small difference of pressure at 7 a.m. between the average of mean values of inland stations and coast stations in England in the same belt of latitude which may be attributed to the dynamical effect of the land upon the flow of air from the westward. The possibility of local coastal gradients deserves further investigation. The tendency of winds to set along the coast line is a phenomenon which has often been remarked upon in the course of the marine work at the Meteorological Office.

[1] *Philosophy of Storms*, p. xiv, *B. A. Report*, 1840, p. 345.

[2] *Met. Zeitschr.*, vol. XIV, 1879, p. 333.

mixing of the surface layers of the air with the upper layers whereby the lower layers acquire momentum at the expense of the upper layers and the qualitative explanation thus provided has been put into a dynamical form which allows of a quantitative estimate of the effect by G. I. Taylor in the paper referred to in note 1, p. 7. The results will be considered later. Here we need only call attention again to the fact that the ratio of the surface-wind to the geostrophic wind is dependent upon the diurnal and seasonal change of temperature of the surface of the ground. The actual cause of the diurnal variation of the wind is the temperature of the ground which undergoes a periodic change with this general physical result, that colder ground means less surface-wind and therefore a smaller fraction of the geostrophic wind, while warmer ground means more surface-wind and therefore a greater fraction of the geostrophic wind. The rule must be universal and is not simply applicable to the case of diurnal variation. Wherever there is a cold surface over which warmer air is passing the cooling of the air by the ground will reduce the factor W/G and conversely the warming of the air by the ground will increase the factor.

We may pursue this line of thought a little further. As long as there is a diurnal variation of wind-velocity without a corresponding variation of gradient we cannot expect agreement between W and G, that is W/G will not become unity; and therefore if we wish to find a region where the wind-velocity will agree with the geostrophic wind we must seek a region where there is no diurnal variation of wind without a corresponding change of gradient. It is possible that such a region may be found somewhere in the upper air.

We have, at present, no satisfactory statistics of the diurnal variation of wind-velocity at high levels in the atmosphere except for observatories at high levels on mountains, the sites of which are themselves subject to diurnal variation of temperature. The number of observations of pilot-balloons is perhaps now numerous enough to supply in some degree the required information but it is not yet reduced to manageable form. The best observations for the purpose are those from the Eiffel Tower summarised by A. Angot, Director of the Bureau Central Météorologique de France[1]. The results which are included in Table I are taken from that summary. They show a diurnal range of wind-velocity with a maximum in the night and a minimum in the day. The nightly maximum is common to all high-level observatories and is already noticeable for light winds though not for strong winds on the anemometer at Potsdam at a height of only 41 metres. It is also indicated at a height of 16 metres in the observations of Hellmann over flat meadow land near Nauen[2].

The minimum of velocity in the upper air in the day corresponding with a maximum of the surface velocity may be attributed to the effect of the eddy-motion which, as explained by Taylor (*loc. cit.* note 1, p. 7), is the real process indicated in the explanation of the diurnal variation of wind-velocity at the surface, and we may suppose that the minimum of the winds on the Eiffel

1 'Études sur le Climat de la France, Régime des Vents,' *Mémoires du B. C. M.*, 1907.

2 Hellmann, 'Ueber die Bewegung der Luft in den untersten Schichten der Atmosphäre,' *M. Z.*, January, 1915.

Tower is due to the dilution of the current by the action of eddy-motion which mixes it with air of less momentum from below. In the night-hours there is at any rate less dilution and at the level of the top of the tower, 305 m. above the ground, probably no dilution at all, so that we may regard the wind at that level (about 1000 ft.) as directly comparable with the geostrophic wind at the same level if we knew it. And, if that be so, then it is clear that in the day-hours the wind at 305 m. over Paris is less than the geostrophic wind and is indeed on the average only about 80 per cent. of the geostrophic wind at 14 h in January, and only 60 per cent. at 9 h in July. It cannot on the average exceed these percentages at the hours named because we have regarded the night values as giving the full equivalent of the gradient. Thus a rise of 1000 feet does not take us above the reach of surface disturbance in the daytime in a region such as Paris. Meanwhile we have a note in the Meteorological Office[1] that Mr S. P. Wing in 1915 found by observations on towers of open work at Ballybunion, Co. Kerry, that the average velocity of the wind at 500 ft. was 90 per cent. of the geostrophic wind at sea level, whereas at 15 ft. the wind was 62 per cent. of the same. We have no record of the hours of the day or the precise dates when the observations were taken, but considering the relative positions the results are not far out of line with the results for Paris.

So far we have drawn no distinction between the various orientations of the isobars and the wind-directions, but these clearly give another element of variation in the relation of the observed wind to the geostrophic wind at the surface which must enter into the consideration of individual observations. Whether or not there be any difference in the ratio from purely meteorological causes such as the increase of gravity for air moving westward as compared with that moving eastward there is certainly a difference due to the peculiarities of exposure in relation to surrounding buildings, trees, etc., and to the general relief of the district.

The relation of the surface-wind W to the geostrophic wind G for different orientations has been examined for a number of localities:

For Falmouth and the neighbouring Pendennis Castle[2], for Pyrton Hill and Southport[3], for Dungeness, Jersey, Holyhead and London (Brixton and Westminster)[4], and recently figures have been obtained, for eight years of observations (1908–1915) at 7 a.m., for fourteen of the telegraphic reporting stations of the Meteorological Office. The comparisons have been made in the first two sets for a miscellaneous assortment of wind-forces but in the others for selected wind-forces. Two stations, Holyhead and Dungeness, appear in Mr Fairgrieve's selection as well as in the larger group of fourteen included in the inquiry at the Meteorological Office and it will be noted that the average percentage of the geostrophic wind is different for both stations, for Holyhead 48 per cent. as against 59 per cent., and for Dungeness, 52 per cent. as against

[1] M. O. Circular, No. 5.

[2] Shaw, *Forecasting Weather* (Constable and Co.), p. 48.

[3] J. S. Dines, *Fourth Report on Wind Structure.* Advisory Committee for Aeronautics.

[4] J. Fairgrieve, 'On the relation between the velocity of the gradient wind and that of the observed wind at certain M. O. stations,' *M. O. Geophysical Memoirs*, No. 9.

58 per cent. The observations are for different groups of years and the differences may perhaps be attributed to changes of observers at the stations. With few exceptions the same kind of difference for different orientations is displayed in both sets of results. The results for these various investigations are given in Table II, p. 20. At some of the stations an anemometer is in operation which has been used either directly or indirectly in the estimations of the velocity of the wind. In these cases the height of the vane of the anemometer is given in a special column of the table. In the other cases, indicated by the letter B in the same column, the velocities of the wind have been derived from estimates in the Beaufort scale converted into velocities by the table given in the Introduction.

The hour of observation, 7 a.m., is selected because for that hour the most complete map is available for the determination of the gradient, a dominant consideration; but a reference to the table of diurnal variation of wind shows that it is not a good hour if the observations of all months are to be combined. Seven o'clock is before sunrise in the extreme winter months and some hours after sunrise in the extreme summer months. The average for that hour therefore gives a composite result which will ultimately require further analysis. It cannot give either the maximum or minimum value, but allowing for one hour's lag in the temperature it may give a serviceable approximation to the mean value for the whole year.

The figures which are given in Table II make no reference to the deviation of the direction of the wind from the run of the isobars. On Taylor's theory the deviation is related numerically to the ratio W/G. The deviation is given in the original results for Southport and Pyrton Hill and also for the fourteen stations of the Meteorological Office. The figures for Pyrton Hill and Southport are given with the diagrams referred to below which illustrate the results for those stations.

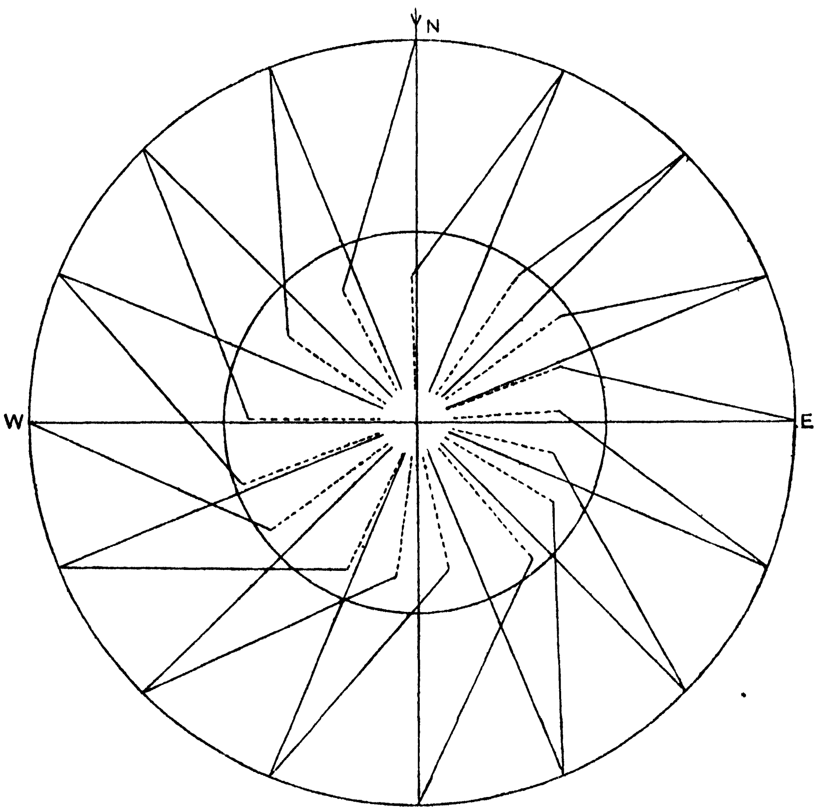
The general meaning of the figures which are given in Table II can be most easily conveyed by plotting those for each station on a diagram in which the velocity of the geostrophic wind is set out as a circle, the radii of which show the orientation of the geostrophic wind. The percentage which the surface-wind for each orientation bears to the geostrophic wind is indicated by a line from the centre to a point within the circle. If the point is properly placed the deviation of the surface-wind from the direction of the isobar can also be indicated. A diagram of this kind may be called a geostrophic wind-rose. Two examples by J. S. Dines representing the relation of the wind to the gradient at Pyrton Hill and Southport, figs. 1 and 2, are taken from the *Fourth Report on Wind Structure* already referred to. The direction and the velocity (as a percentage of the geostrophic wind) are indicated by the dotted lines, the extreme points of which are connected with the geostrophic winds, with which they correspond, by lines which represent the vector-differences of the two.

The diagrams are necessarily somewhat complicated because so many items have to be expressed. A simpler form of geostrophic wind-rose showing only the ratio of W/G without any indication of the deviation of direction is

FIG. 1. COMPARISON OF GEOSTROPHIC AND SURFACE-WINDS AT PYRTON HILL.

Mean surface wind-velocity = 42·6 per cent. of geostrophic.
Mean deviation of surface from geostrophic wind = 32°.

Geostrophic wind-direction	WNW	NW	NNW	N	NNE	NE	ENE	(1)
Deviation α	42°	45°	34°	29°	24°	11°	13°	(2)
W/G, calculated % ...	7	0	27	39	50	79	75	(3)
W/G, observed %	49	44	40	39	38	48	48	(4)



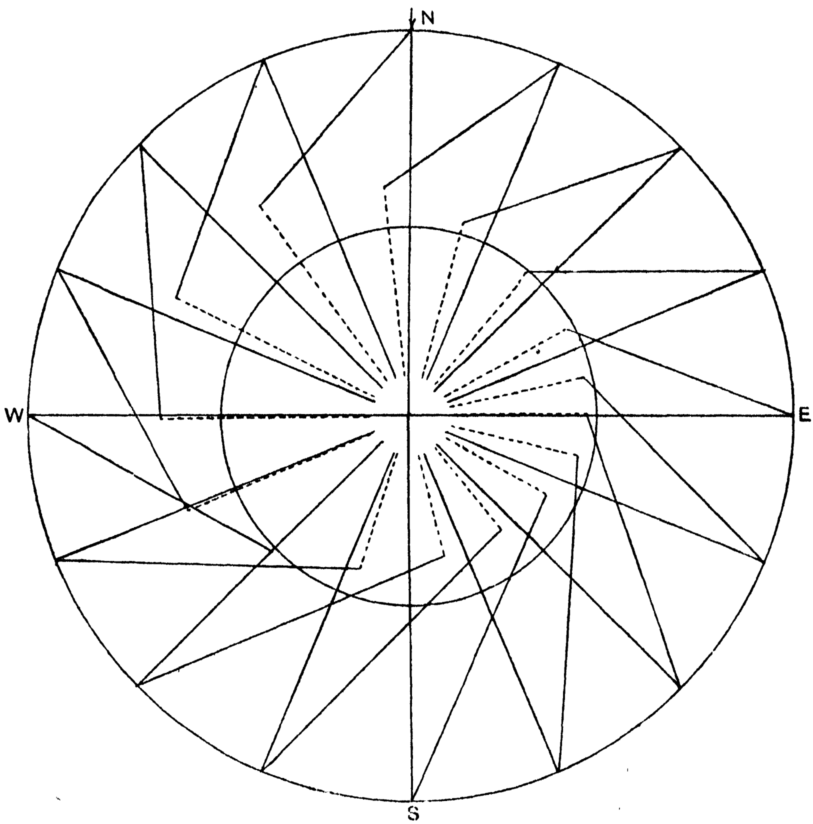
(1)	W	WSW	SW	SSW	S	SSE	SE	ESE	E
(2)	37°	43°	37°	35°	40°	37°	31°	27°	22°
(3)	20	5	20	25	12	20	34	44	45
(4)	47	43	42	40	48	42	37	37	40

Note. The explanation of the calculation of the ratio W/G, of the surface-wind to the geostrophic wind, from the deviation α is given in chap. IV of this Part. The calculation fails altogether when the angle of deviation amounts to more than 45°. It is noteworthy that the deviation of 45° is equalled or sur-

FIG. 2. COMPARISON OF GEOSTROPHIC AND SURFACE-WINDS AT SOUTHPORT.

Mean surface wind-velocity = 51·6 per cent. of geostrophic.
Mean deviation of surface from geostrophic wind = 44°.

Geostrophic wind-direction	WNW	NW	NNW	N	NNE	NE	ENE	(1)
Deviation α	46°	45°	41°	36°	31°	30°	29°	(2)
W/G, calculated % ...	—	—	10	22	34	37	39	(3)
W/G, observed % ...	46	45	41	69	60	53	49	(4)



(1)	W	WSW	SW	SSW	S	SSE	SE	ESE	E
(2)	45°	49°	59°	62°	61°	56°	46°	36°	30°
(3)	—	—	—	—	—	—	—	22	37
(4)	51	42	38	38	41	45	47	47	48

passed at Southport for the nine orientations S.E. to N.W., a line which is approximately transverse to the coast line at Southport. The results point to a refraction of the isobars in crossing the coast which is most pronounced with winds along the coast from sea inwards. (See chapter VIII.)

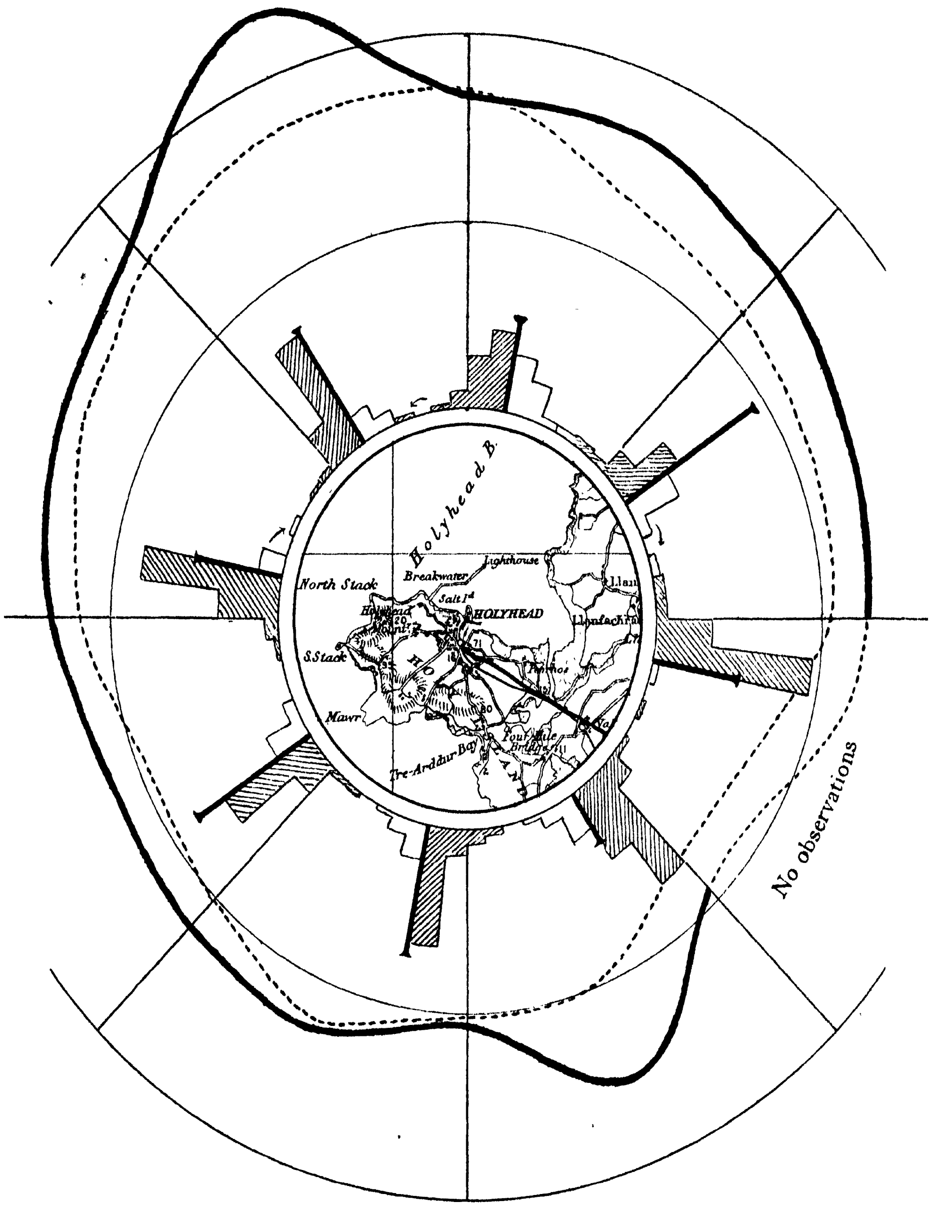
used for representing the results for Falmouth and Pendennis Castle in the work referred to and also by J. Fairgrieve in his Report. In these cases a continuous line is drawn connecting the points representing the percentage of the geostrophic wind for the several orientations of the geostrophic wind which shows at once the comparative values of the relation for the orientations of the site.

In the work on the rest of the stations included in the table endeavour has been made to present all the facts to the reader by giving the percentage of the geostrophic wind for different orientations in the form of a continuous line drawn within a circle representing the geostrophic wind and surrounding a reproduction on a reduced scale of a circular portion of the ordnance map of the district on the scale of four miles to an inch, which gives the reader an idea of the features of the geographical relief of the land in the immediate neighbourhood of the observing stations. Information as to the deviation of the surface-wind from the geostrophic wind is given in the form of columns representing percentage frequencies of deviations of given amount arranged in groups, for steps of two points, on either side of a middle group embracing four points, namely two points of veer and two of back. The percentage number of cases in this central group is indicated by the length of a pin-shaped mark on the scale of one inch of length (shown by the distance between two consecutive circles in the figure) to 50 per cent. The percentage frequency of the other groups is shown on the same scale by the length of the respective columns, the shaded columns indicating the surface-winds which are *backed* from the geostrophic wind and the unshaded columns those which are *veered*. The combination of a veer and back of two points within the central group may be held to give an artificial prominence to that group, but the classification in these cases with estimated winds is not very precise and the veering of the surface-wind from the geostrophic wind can only be looked upon as due to some local peculiarity in the determination of the wind. Hence the portion of the diagram on the right-hand side of the central pin may be regarded mainly as an indication of the peculiarity of the site in that respect. In these frequency diagrams winds of all forces are included.

The percentage relation of the velocity to the geostrophic wind, for surface-wind estimated as force 4, is given by a dotted line and the corresponding results when the surface-wind is estimated as force 6 (which are not included in Table II) by a full line. The diagrams for Holyhead and Yarmouth are reproduced here. Regarding for the moment the results for force 4 the former shows very notable difference between the small percentages for winds from the South-Eastern region coming over the Welsh mountains a long way off, and the winds from the open sea in the opposite quarter. At Yarmouth the same kind of feature is characteristic of the results but in this case it is the Easterly side which gives winds with a closer approximation to the gradient as compared with the winds in the Western quadrants which come overland.

The results for force 6 are rather disturbing. At Holyhead, contrary to

FIG. 3. RELATION BETWEEN GEOSTROPHIC AND OBSERVED SURFACE-WINDS AT HOLYHEAD.



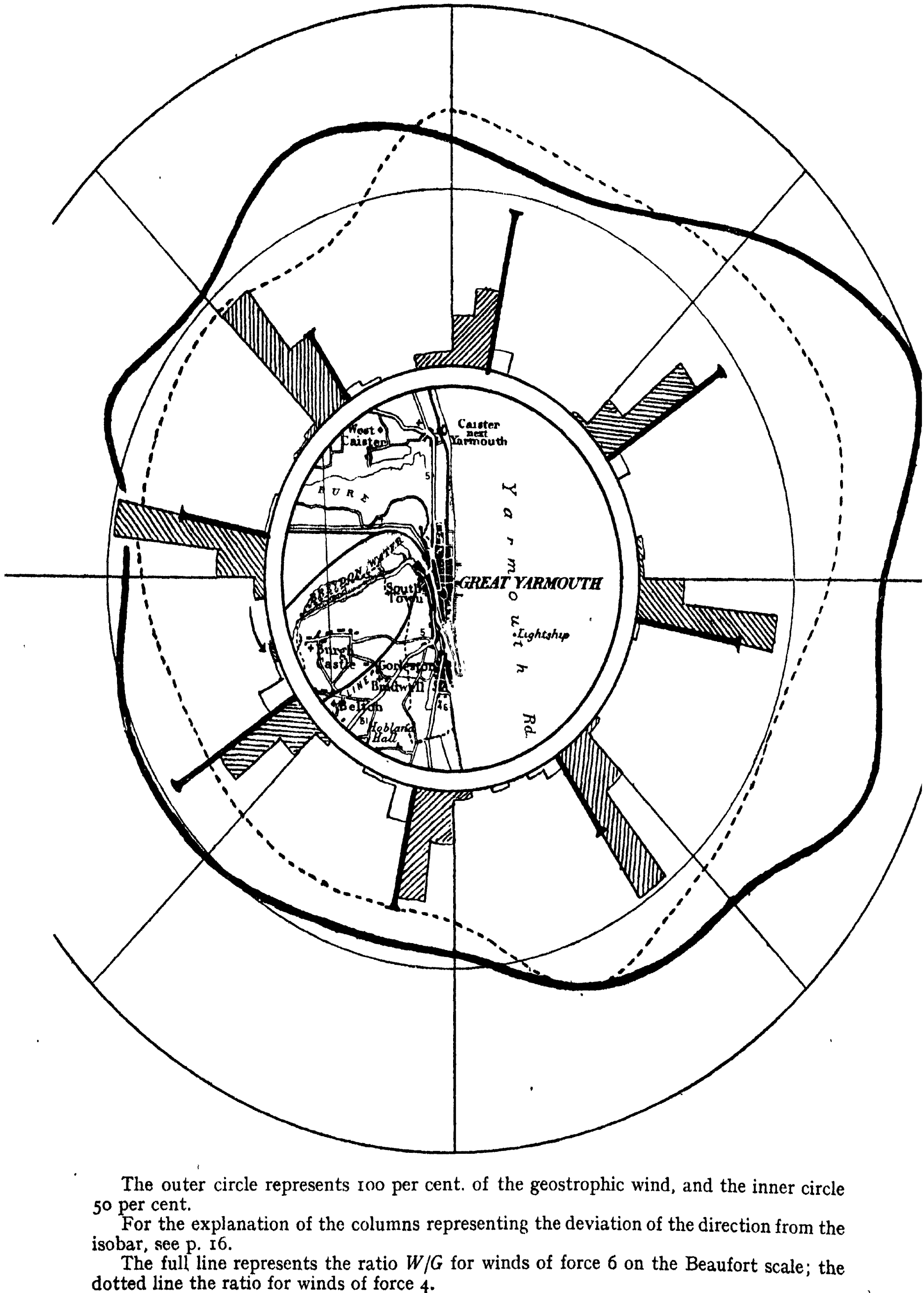
The outer circle represents 100 per cent. of the geostrophic wind, and the inner circle 50 per cent.

For the explanation of the columns representing the deviation of the direction from the isobar, see p. 16.

The full line represents the ratio W/G for winds of force 6 on the Beaufort Scale; the dotted line the ratio for winds of force 4.

Radius of Map = 4 miles.

FIG. 4. RELATION BETWEEN GEOSTROPHIC AND OBSERVED SURFACE-WINDS AT GREAT YARMOUTH.



The outer circle represents 100 per cent. of the geostrophic wind, and the inner circle 50 per cent.

For the explanation of the columns representing the deviation of the direction from the isobar, see p. 16.

The full line represents the ratio W/G for winds of force 6 on the Beaufort scale; the dotted line the ratio for winds of force 4.

Radius of Map = 4 miles.

expectation with a single exception, they show a greater percentage of the gradient than those of force 4. At Yarmouth the excess is less marked though it exists for many directions. There are not sufficient examples of force 6 at Holyhead to complete the circle and the numbers of observations for other orientations are not enough for a satisfactory conclusion; but when the diagrams for the other stations are examined there is more unanimity of opinion in favour of a higher percentage with force 6 than with force 4 than can be explained by lack of observations. The explanation probably lies in a tendency on the part of the observers to over-estimate the stronger winds and this explanation seems also to explain the high estimate of wind from the N.N.W. at Holyhead which goes actually beyond the equivalent of the gradient.

We have already pointed out that in dealing with the relation of surface-wind to the gradient we have to rely upon inadequate measurements for lack of better. As the material of observation improves by the introduction of suitably exposed anemometers and in other ways it will become possible to revise the crude results which at present are all that we have to offer. Even at the worst we are better off than our colleagues who have to deal with the winds in the interior of continents, for which the distribution of pressure as plotted on the map refers to a hypothetical region at sea level far below the position where the observations of wind are made.

In general with regard to the relation between the surface-wind and the gradient over the land we may conclude from the considerations set out in this chapter that there is no ground for surprise if, on any occasion, the surface-wind when expressed as a fraction of the geostrophic wind be found to range a long way from unity and large divergence in direction is equally possible while the upper wind still keeps within the first law of atmospheric motion. The effect of friction makes no provision for surface-wind velocities in excess of the geostrophic wind. The suggestion may be put forward that the only cases of the kind that occur are those in which the katabatic effect adds to the velocity of the surface-wind by a gravitational flow in the direction of the isobar, and this possibility should be borne in mind because it may prove to be an explanation of certain local winds of considerable violence such as the bora of the Adriatic, the violent winds in the fiords of Norway and Iceland which may be noticed sometimes in the charts of the Daily Weather Report; also the winds on the East and West coasts of Greenland[1] and the extremely violent gales experienced on the shores of the Antarctic by the expedition under Sir Douglas Mawson[2].

[1] W. H. Hobbs, 'The Rôle of the Glacial Anticyclone in the Air Circulation of the Globe,' *Proc. Amer. Phil. Soc.*, vol. LIV, No. 218, 1915.

[2] Sir Douglas Mawson, D.Sc., B.E., *The Home of the Blizzard*, chap. VII. Heinemann, 1915.

Note. A recent discussion of the diurnal variation of the wind in altitude (a minimum in the day with backing and a maximum in the night with veering) is given by G. Reboul in *Comptes Rendus*, vol. CLXVI, p. 295, 1918.

TABLE II. THE RELATION OF SURFACE-WIND TO GEOSTROPHIC WIND FOR A NUMBER OF STATIONS IN THE BRITISH ISLES

The Values of W/G expressed as the Percentage ratio of the Surface-Wind to the Geostrophic Wind for 16 points of the compass. The highest percentage ratio for each station is indicated by the symbol (:) the lowest by (-).

	No. of years' obsn	Forces comp'd	Height of vane or cups above ground	Nature of site	Height of ground	N 0°	NNE 22½°	NE 45°	ENE 67½°	E 90°	ESE 112½°	SE 135°	SSE 157½°	S 180°	SSW 202½°	SW 225°	WSW 247½°	W 270°	WNW 292½°	NW 315°	NNW 337½°	Mean of all
			ft.		ft.	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%	%
Falmouth	1	all	41	Between cliff & harbr.	167	36:	36:	33	30	31	32	32	31	29	28-	29	33	34	35	34	36:	32
Pendennis Castle	1	all	65	Conical headland	256	58	66	74	83	85:	83	74	69	63	58	54	52-	53	56	54	54	65
Pyrton Hill	2	all	98	Slope: hills to plain	500	39	38	48	48	40	37-	37-	42	48	40	42	43	47	49:	44	40	43
Southport	5	all	62	Flat sand shore	18	69:	60	53	49	48	47	47	45	41	38-	38-	42	51	63	65	69	52
Dungeness	12	12 m/s	B	Spit of shingle	21	49	57	70:	68	61	56	48	48	49	53	56	48	41	39	38-	46	52
Jersey	12	12 m/s	B	Railway station	25	54	55	56	54	56	48	46	47	47	47	48	56	57	50	49	56	52
Holyhead	12	12 m/s	B	Flat island	15	81:	75	44	43	35	17	14-	34	39	49	48	40	48	60	68	80	48
London, Brixton	4	12 m/s	B	Town garden	77	28	29	36	38	31	17	17	23	24	25	24	23	23	20	23	—	23
,, St James's Pk.	6	12 m/s	B	Town park	27	25	29	29	29	24	13	11	12	13	23	24	22	18	18	24	19	21
Shetland, Lerwick	4	4	B	Low cliff	54	44	70:	52	61	50	57	48	47	43	40-	40-	44	45	53	45	52	49
,, Dunrossness	4		B	Flat ridge	112																	
Stornoway	8	4	B	Sloping shore of bay	51	48-	57	63:	58	51	52	48-	49	60	57	52	51	50	48-	49	50	53
Castlebay	8	4	B	Rocky island	37	50	49	48	48	48	50	58:	52	53	50	50	46-	51	57	53	54	51
Malin Head	8	4	B	Headland knoll	208	72	73:	60	39	65	66	51	44-	46	48	48	56	57	67	70	68	58
Blacksod Point	8	4	B	Low shore / Low ridge	37 / 330	56	69	78:	54	64	58	56	52	54	54	47	45-	45-	48	48	53	55
Cahirciveen	8	4	45	Mouth of glen	30	58	57	55	47	55	42	39	38	37-	40	41	54	54	60:	59	57	50
Holyhead	8	4	B	Flat island	15	85	68	59	57	53	37	34-	49	51	61	51	48	60	69	79	87:	59
St Ann's Head	8	4	B	Level headland	140	64	61	51-	63	55	65	59	56	54	55	57	61	64	63	66:	59	60
St Mary's, Scilly	8	4	32	Hilly island	118	74:	63	59	59	67	70	59	47	51	45-	49	49	54	63	65	61	58
Portland Bill	8	4	B	Low point of headland	19	65	68	73	80:	68	56	53	53	48-	51	56	59	64	67	71	73	63
Dungeness	8	4	B	Spit of shingle	21	62	64	69:	68	61	55	61	52	56	52	57	51	49-	49-	67	62	58
Aberdeen	8	4	74	College roof	46	35	45	61:	57	51	45	43	35	36	32	32	31-	39	46	54	43	43
Spurn Head	8	4	40	Spit of sand	26	80:	85	64	72	65	72	64	54	49-	49-	49-	53	59	76	76	73	65
Yarmouth	8	4	40	Spit of sand	13	72:	62	66	72:	62	51	54	63	38	37	36	34-	39	43	46	43	51
Paisley	8	4	B	Inland station	—	37	33	53:	53	47	33	28-	35	33	35	36	36	41	36	42	47	38
Carnforth	8	4	B	,, ,,	—	30	37	35	40	43:	39	28	25-	30	35	32	31	31	36	43:	—	33
Belvoir Castle	8	4	B	,, ,,	—	50	59:	59:	44	50	58	54	50	44	36-	37	37	41	40	43	36-	43
Geldeston	8	4	B	,, ,,	—	30	30	32	—	42:	40	36	31	29	26	29	29	29	29	30	21-	30
Woburn	8	4	B	,, ,,	—	49	49	54	57	40	45	61:	42	31-	43	43	44	49	50	49	46	46

CHAPTER III

The relation of the surface-wind to the gradient over the sea

THE available observations at sea are exclusively estimates according to the Beaufort scale and very few direct comparisons have been made between the wind and the corresponding gradient. The best prospective material for the purpose is perhaps that contained in two of the publications of the Meteorological Office, namely, the Synchronous Weather Charts of the North Atlantic and adjacent continents, 1st August, 1882, to 3rd September, 1883, which were published in 1886, and the Synchronous Charts of the North Atlantic illustrating the stormy period of the winter of 1898–99 which were published in 1900 but are now out of print[1]. An inspection of these charts, as of any other synchronous charts of any part of the ocean, confirms the view that close isobars are accompanied by strong winds, but so far as is known no direct comparison has been made between the numerical values of the wind-velocity and the gradient. It should be noticed that in these charts, as in all the charts issued by the Office before 1911, the values of the pressure are not corrected for the variation of latitude; but this makes little difference for the middle latitudes of the North Atlantic, which offer the best examples for comparison, because they are not far from the datum latitude of 45°.

The maps of the regions of the British Isles which are included in the Daily Weather Report and from which geostrophic winds for certain regions are regularly computed do not afford a satisfactory measure of the gradient over the sea on the West, because the isobars cannot be carried beyond the Western shores. The land-locked seas, the North Sea, the Irish Sea and the English Channel can be bridged by isobars drawn according to the pressures on either side. In those cases however the span of the bridge is rather long and observations from those seas are comparatively rare. A series of such observations for the North Sea has, however, recently become available. They are made according to ships' time in accordance with the rules for observations at sea and consequently refer to 8 a.m., noon, 4 p.m. and so on, whereas the maps to which they must be referred are for 7 h or 13 h or 18 h. Hence the comparison is at the best somewhat vague. Accordingly in dealing with the comparison of observations with computed gradients the method of frequencies has been employed and the *mode* or group of maximum frequency has been taken as giving the most probable value of the relation of surface-wind to gradient.

In the examination of the comparison at the Meteorological Office[2] the winds of different strength have been considered separately, light winds with

[1] The publication of Daily Synchronous charts of the Atlantic was begun by Captain Hoffmeyer, Director of the Danish Meteorological Institute, in 1873 and has been continued by that Institute in co-operation with the Deutsche Seewarte, Hamburg.

[2] *Report to the Hydrographer*, M. O. No. 4977, 13th April, 1918.

geostrophic velocity below 8·5 m/s, moderate and strong—with geostrophic velocity between 8·5 m/s and 18 m/s—and very strong when the geostrophic velocity exceeded 18 m/s. The observations have also been grouped separately for direction and velocity. For direction the groups have been made according to the number of "points" in the veer of the geostrophic wind from the surface-wind, a point being $11\frac{1}{4}°$; and for velocity according to the ratio of the surface-wind to the geostrophic wind, the limits selected for the ratios being ·24 to ·36, ·36 to ·48, ·48 to ·60, ·60 to ·72, ·72 to ·84, ·84 to ·96 and ·96 to 1·08, so that the mean values for each group range approximately about ·3, ·4, ·55, ·65, ·8, ·9 and 1. The winds for the different quadrants have also been dealt with separately. The results for the moderate and strong winds are given in the following table.

Relation of surface-wind to the geostrophic wind (between 8·5 m/s and 18 m/s) over the North Sea.

A. Direction: Percentage frequency of points in the veer of the geostrophic wind from the surface-wind.

Deviation in points ...	−4	−3	−2	−1	0	+1	+2	+3	+4	+5	+6	+7	
	%	%	%	%	%	%	%	%	%	%	%	%	
NW quadrant	0	1	0	2	21	32	**33**	14	1	0	0	0	(104)
SW ,, ...	0	2	4	0	23	**30**	19	14	7	2	2	0	(103)
SE ,, ...	2	2	2	2	16	**29**	16	**19**	9	1	1	0	(99)
NE ,, ...	0	2	2	0	2	**31**	27	22	3	0	3	3	(95)

B. Velocity: Percentage frequency of the ratios of surface-wind to geostrophic wind within assigned limits.

Limits of ratio	·24–·36	·36–·48	·48–·60	·60–·72	·72–·84	·84–·96	·96–1·08	
NW quadrant	1	5	16	**38**	29	10	0	(99)
SW ,,	8	16	**34**	29	12	2	1	(102)
SE ,,	13	19	**26**	18	18	2	**4**	(100)
NE ,,	9	**20**	9	**29**	20	6	**9**	(102)

Note. The differences between the total numbers in each row and 100 arise from the casting up of fractional percentages in the several cases. The "modes" are shown by thick type.

In considering these results we have to remember the limitations of the comparison, the difference of the time of observation, the liability to error in the estimation of the direction and force of the wind at sea, the uncertainty in the determination of the local gradient and the absence of any allowance for the curvature of the isobar. To these causes we may attribute the great width of the range of the relation both in direction and velocity. We may remark that we are prepared to accept positive deviation from the isobars up to 8 points or 90° but we have no reason to give for negative values of the deviation of the geostrophic wind from the surface-wind and further inquiry is needed before

a definite opinion can be formed of their reality. Nor can we, at present, give any satisfactory reason for surface-winds which are estimated to be of the full velocity of the geostrophic wind or beyond it, of which the Easterly quadrants seem to afford a substantial number of examples. We find everywhere a tendency for observers to form high estimates of winds in the North East quadrant, so the occurrence of higher estimates in that quadrant is not surprising, but there is not sufficient material to form a satisfactory opinion as to the true meaning of that experience. We know that the force of gravity upon any body moving Westward is greater than that upon the same body moving Eastward because the centrifugal effect due to the rotation of the earth is diminished in the one case and increased in the other, but the difference is hardly large enough to show in a rough investigation of this kind.

But taking the figures in the table as we find them we may note that for the winds in the North West quadrant the frequencies group themselves with a fair approach to symmetry round a mode of about ·67 of the geostrophic wind for velocity, with a veer of about $1\frac{1}{2}$ points or say 18°; and a similar statement holds for the winds in the South West quadrant with a mode of about ·55 of the geostrophic wind and a veer of one point or 11°. No such approach to symmetry of arrangement is shown for the two Eastern quadrants. The winds in the North East quadrant show two distinct modes with ratios of about ·4 and ·7 and perhaps a third about unity, and the deviations are divided between one point, two points and three points. Those in the South East quadrant show a mode with a ratio of about ·55 with the definite suggestion of another at ·8 and a hint of a third about unity, while in the table for direction there are modes at one point (11°) and three points (35°).

There is not enough material here upon which to found a theory but it is not out of place to remark that the conclusions which the tables suggest are what we might fairly expect from the considerations which have been set out in our discussion of the influence of surface-temperature upon the relation of the wind to the gradient over the land. We have seen that when the surface is relatively cold and is therefore absorbing heat from the air which passes over it, as in the night-hours, the ratio of the surface-wind to the geostrophic wind is diminished; whereas, on the contrary, when the surface is relatively warm and is therefore supplying heat to the air which is passing over it, as in the day-hours, the ratio of the surface-wind to the geostrophic wind is increased. In the open sea there is no appreciable diurnal variation of temperature of the water which forms the surface and consequently no diurnal variation in the relation of the wind to the gradient, but instead of that we have more or less permanent differences of temperature between the water surface and the body of air which flows over it which must have their effect upon the relation of the surface-wind to the geostrophic wind.

Looking at the distribution of isotherms of the water in the North Sea[1] we may conclude as a general rule that winds in the N.W. quadrant generally pass from colder to warmer water and winds in the S.W. quadrant from

[1] *The Weather of the British Coasts*, M. O. Publication 230, chap. XII. 1918.

warmer to colder water, and hence the winds of the North West quadrant ought to approach the geostrophic wind more nearly than the winds of the South West quadrant. This may account for the difference of the mode of ·67 of the geostrophic wind for the North West and ·55 for the South West quadrants, though the difference between the deviations from the isobars being 18° for the North West and 11° for the South West would still require explanation.

The same ideas give some clue to the apparently erratic behaviour of the winds in the Eastern quadrants. Winds coming from the North East have also to pass over the water with its variations of temperature, as likewise have the winds which come from the South East, but the great land area of the globe lies immediately to the East and South East of the North Sea and the temperature of the body of air which passes over the North Sea from the Eastward is controlled by the land, whereas the body of air which comes from the Westward is very little affected by the land if its course is from the North West, and even if it comes from the South West it is not so much affected as that which comes from Eastern quarters.

The vicissitudes of the latter are known from the experiences of our climate to be extremely varied. From the East we get the hot spells of summer and the cold spells of winter and in all continental countries there is a great range of temperature between day and night. These vicissitudes will have left their mark on the air that is launched from the Eastern shores of the North Sea, and hence the relation of the temperature of the air to the temperature of the sea over which it passes will vary on different occasions between the extremes of much warmer and much colder. The relation to the gradient will therefore naturally show, according to the occasion, the one or the other of the modes which we have recognised for the winds from the North and the South in the Western quadrants.

Thus we cannot look for complete simplicity of relation of the surface-wind to the gradient even over the sea though in the open ocean the causes of variation may be greatly simplified because the régime of the temperature is free from the complications which affect our land-locked seas.

To complete the information which has been derived from the observations of wind and gradient over the North Sea we may add some notes on the more detailed tables in which the winds of different strengths are treated separately. They are as follows: In the case of the North West quadrant the winds are at a veer of one or two points and for velocity uniformly in the group about ·65. For the South West quadrant the groups of different strength all show a veer of one point; as regards velocity the lighter winds have a mode about ·65 and the stronger about ·55; there is one secondary mode for moderate winds about ·4.

In the South East quadrant the veer is mostly one point, but for strong winds there is a secondary mode at three points; for velocity there are many secondary modes which in three instances are at two groups apart. In the North East quadrant, for direction the veer increases with the velocity of the wind, for velocity the distribution of the modes is very irregular.

These notes show some suggestion of a closer agreement between the surface-wind and the geostrophic winds with lower velocities as the theory to be discussed later would indicate, but more material is needed before a satisfactory opinion can be formed. An apology is indeed owed to the reader for dealing with so important a subject as the relation of surface-wind to gradient over the sea in so inadequate a manner, but the importance of the subject is itself the excuse for making what use is possible of the material that is at hand in the hope that more may be forthcoming.

CHAPTER IV

The variation of wind with height in the surface layers

By way of giving a practical basis to the ideas which we propose to develop in this chapter let us set ourselves to face the question "Is it possible, from a measure of the wind-velocity by a fixed anemometer or by an instrument held in the hand, or from an estimate on the Beaufort scale on land or sea, to obtain working measures of the wind-velocity in the layers immediately above the observer and if so, up to what heights may the calculation be extended with reasonable accuracy?"

Considering the last part of the question we may recall the conclusion drawn in the Introduction from W. H. Dines's table of correlation between deviations from the normal pressure and temperature in the upper air, namely, that at all levels from three kilometres to nine kilometres in all seasons of the year the correlation between the deviations of pressure and temperature is very close, generally above ·75 and always positive, whereas at the level of two kilometres the correlation though still equally high for the winter half-year, from October to March, is only of the order of ·5 for the summer half-year. Below that level at one kilometre it is of the order of ·5 for the winter half-year and ·3 for the summer half; and at the surface there is no correlation at all. Anticipating to some extent what follows we may attribute the interference with the direct correlation, in part at least, to the turbulence of the motion in the surface layers caused by the friction with the land or water over which the air passes. The effect of the turbulence, which causes mixture of the layers above with those beneath, diffuses upwards at all seasons of the year but especially in the summer half when the dynamical effect is exaggerated by the transference of heat from the ground to the air in contact with it. The region indicated by the expression "the surface layers" may therefore be regarded roughly as having a thickness between one and two kilometres in the winter half-year and between two and three kilometres in the summer half-year. Above these levels we may contemplate a separate régime of winds to be treated provisionally as independent of the turbulence due to the surface. We have already seen that the air at the top of the Eiffel Tower is affected by turbulence in the winter and still more so in the summer, so that we must not be surprised to find the influence extending to a kilometre or more.

In endeavouring to form a mental picture of the régime of winds in the lower layers on these lines the natural order would be to start from the undisturbed wind and consider the surface-wind to be connected therewith by a formula which can be represented by a diagram showing the relation of wind with height. It is fair to assume that in such a diagram the wind would have

its least velocity at the surface and increase upwards until it reached the undisturbed wind; if the undisturbed wind be assumed to be the geostrophic wind for its own level, further changes at higher levels would correspond with changes in the geostrophic wind at successive levels and such changes, depending on the gradual change in the gradient due to the distribution of density of the air at successive levels, would necessarily be very gradual except in extraordinary meteorological circumstances.

We have already seen that for the same undisturbed or geostrophic wind in the upper levels the corresponding surface-wind will certainly have different values for different times of the day, for different seasons of the year, for different localities in respect of geographical relief, and for different azimuths; consequently a series of curves will be required to connect the undisturbed wind with the surface-wind. The portions of the curves which we can plot from observations in the lowest layers are the tail-ends of curves the special forms of which are dependent upon the particular conditions of the time. It is practically hopeless to suppose that any general formula can be devised to be applied in all cases to give the variation from the surface up to the undisturbed wind and consequently the approach to the solution of the question by defining empirically the relation of the surface-wind to the wind in the successive layers immediately above the surface must necessarily consist of the numerical equivalents of a series of diagrams which cannot be immediately coordinated.

The question is further complicated by the variations in the direction of the wind at different levels which are associated with the observed changes in velocity. On the general principle that the retardation of any layer of moving air destroys the power of that layer to maintain its balance with the distribution of pressure it is clear that the retarded layers of air near the surface will be deviated from the path of the undisturbed current above them by yielding to the pressure-gradient which their velocity cannot balance and rearranging themselves with a component of motion towards the side of lower pressure which will be greater the greater the departure of the velocity from the measure required for balance. In ordinary circumstances there is a deviation of some 20° to 30° between the direction of the surface-wind and that of the geostrophic wind, or that of the lower clouds which is generally in close agreement with the line of the isobars, but in this matter again the results are dependent upon meteorological and local conditions and in some cases the deviation of the surface-wind from the line of the isobars is much greater[1].

We must look to some general theory, if we can find one, to give us the clue to the coordination of all these variations, and in the meantime we will place on record the results of observation as a basis and test of future theory. For the present we will confine our consideration to the variations of speed of the wind with height. We should be glad to begin with the variations of wind near the surface of the sea but the only direct observations that we can recall are

[1] J. S. Dines, *Fourth Report on Wind Structure*, Advisory Committee for Aeronautics. Report No. 92, p. 19, 1913.

those of G. I. Taylor[1] on the *Scotia*, who found a very irregular ratio of the velocity at 70 ft. (21·5 m.) to that at 45 ft. (13·7 m.) with a probable mean of 1·07 which is in close agreement with the scale for anemometers up to 30 metres quoted below.

It is usual to express the variation of wind-velocity at different levels as a fraction of the value at one of the levels. Thus the Meteorological Office[2] gives the following for the wind at various heights above open grass-land up to 30 metres as a fraction of the velocity at 10 metres which is taken as representing the normal height and exposure of the vane of a tube-anemometer.

Height in metres ...	0·5	1	2	3	4	5	10	15	20	25	30
Ratio of velocity of wind to that at 10 m. ...	·50	·59	·73	·80	·85	·89	1·00	1·07	1·13	1·17	1·20

And for the layers quite close to the ground observations by G. I. Taylor and C. J. P. Cave not yet published gave the following result:

		Height in feet		
		1·25	4	6
Ratio of velocity to the velocity at 6 ft.	over grass 1 in.–6 in. high	·65	·86	1·00
	over short cropped grass 2½ in. high	·71	·87	1·00
	over pond with small waves	·82	·90	1·00

The ratio of decrease appears to be independent of the magnitude of the velocity but dependent upon the nature of the surface. When the ground is flat the projections being blades of grass or small plants the decrease is also independent of the direction of the wind, but in the case of ploughed fields or trenches, where furrows run in one direction only, the direction of the wind will make a difference.

In discussing the variations of wind with height obtained in the series of observations with kites at the station maintained for the University of Manchester at Glossop Moor[3], Miss Margaret White came to the conclusion that the variations could be represented better by invariable additions to the wind recorded at the surface than by an increase proportional to the velocity at the datum level, but this result is probably due to the selective character of the winds necessary for raising kites: it is certainly not applicable to many cases of observations with pilot-balloons.

The observations on the Eiffel Tower at 305 m. as compared with those on the tower of the Bureau Central Météorologique at 21 m. above ground give the following results. The velocities are given in metres per second.

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Year
Eiffel Tower	10·2	9·8	9·4	8·7	8·3	7·6	7·4	8·0	8·2	9·3	9·2	9·9	8·83
Bureau	2·4	2·5	2·5	2·4	2·2	2·2	2·0	2·0	1·8	1·8	2·0	2·2	2·17

[1] *Scotia Report*, p. 65. [2] *Annual Summary of the Monthly Report*, 1916.
[3] Unpublished memoir communicated to the Meteorological Office.

Hellmann[1] has compiled observations over flat meadow-land at Nauen at heights 2 m., 16 m. and 32 m. above ground from which he confirmed an empirical formula $v = kh^{\frac{1}{5}}$ which is not very different from the formula $v = kh^{\frac{1}{4}}$ suggested by Archibald[2] from observations with kites in 1888, which was found to be in accord with Vettin's results[3] for the motion of clouds at much greater heights. The formula is in good agreement with J. S. Dines's observations, to be referred to later, for moderate winds in the middle of the day; but the increase of velocity is not sufficiently rapid for cloudy weather or for the early morning; and, from the nature of the case, it is probable that a logarithmic formula is more likely to be applicable than one depending on a single power of the height.

Passing on to observations for levels accessible only by aircraft or pilot-balloon the most instructive observations for our immediate purpose are those of J. S. Dines[4] carried out at South Farnborough, in October and November, 1912, with which are included other observations made previously at Pyrton Hill. Two theodolites were used in almost all cases. His conclusions are best represented by the diagrams which accompany his report and are accordingly reproduced here.

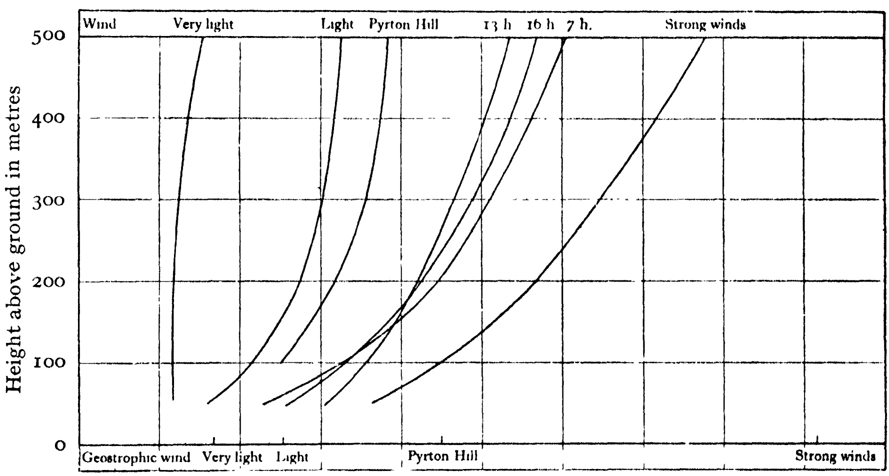


FIG. 1. Change of Wind-Velocity with Height within 500 metres.

The numbers of observations upon which the curves are based are not numerous but they show some noteworthy results. The first diagram represents the variations of wind-velocity with height up to 500 metres for different groups of winds selected according to their velocity at 500 metres, namely very light winds when the velocity at that level was 4 m/s or less, light winds more than 4 m/s and not more than 10 m/s and strong winds with a velocity greater than 10 metres per second at that level. Another curve is added

[1] *Meteorologische Zeitschrift*, 1915.

[2] *Nature*, vol. XXVII. p. 243.

[3] Vettin, *M. Z.*, vol. XVII. p. 267, 1882.

[4] *Fourth Report on Wind Structure.*

representing the combination of ascents at Pyrton Hill. The geostrophic wind is marked for each of the groups. The shape of the curve for very light winds is peculiar but it tends to confirm the suggestion put forward in the preceding chapter that light winds at the surface tend to show closer accordance with the gradient than stronger winds.

With the original diagram for these several groups of winds we have incorporated the curves of another diagram in the same report which shows the result of grouping the observations according to the time of day for the purpose of disclosing the diurnal variation of wind-velocity. The effect is quite conspicuous. In the early morning at 7 h the velocity is least, but the curve shows greater increase near the ground, so that at less than 100 metres the morning wind becomes weaker than the afternoon wind, marked as 16 h; the midday wind for 13 h, though much the strongest of the three at 50 metres, has a smaller fractional increase with height and becomes the least strong at less than 200 metres. At what level precisely this reversal takes place on any particular occasion is not disclosed, but the reversal is a characteristic phenomenon which is exhibited by the maximum in the night and minimum in the day shown by the records of wind at all high-level stations.

In the same report the variations of wind-velocity with height on cloudy and cloudless days are compared though only for a few occasions. The results show that the velocity increases from the surface more rapidly on cloudy days than on cloudless days and hence cloudiness as compared with freedom from cloud has the same kind of effect as the early morning compared with midday, and both owe their influence to the difference in the conditions of the warmth of the surface relatively to the air above it which affects the turbulence of the surface layer.

These different types of curve representing the variation of the velocity of wind with height should be borne in mind as carrying the key to differences between the various kinds of exposure, and hence to the differences to be expected at different stations.

When examining with Captain Cave the results of his observations with pilot-balloons at Ditcham Park in 1910 we noticed that as a general rule in the diagrams plotted for velocity against heights above sea level the lowest pair of observations formed a tail to the curve which pointed with great fidelity to the origin. From that we concluded that at the beginning of the ascent the velocity was not infrequently directly proportional to the height above sea level so that the velocity V at height H above ground would be given by the formula

$$V = \frac{H + a}{a} V_0,$$

where V_0 is the velocity given by a well-exposed anemometer and a is a constant for the particular site. The value of a for Ditcham Park in Hampshire would be 167 metres or 550 feet, the height above sea level. On examining the corresponding observations for Pyrton Hill in Oxfordshire, which is at 500 feet above sea level, the same relation was found to hold generally; it was

supported by observations at Brighton and less satisfactorily by observations at Glossop Moor, Derbyshire, which is at a higher level.

It was on account of this curious empirical result that the formula of velocity proportional to height above sea level until the geostrophic velocity was reached was put forward as a rough working rule in the writer's report on Wind Structure[1] to the Advisory Committee for Aeronautics. And it is reproduced in *Forecasting Weather*, p. 351. It is too empirical to find a permanent place in scientific literature, and from what has been said previously in this chapter it is clear that it cannot hold at all hours of the day but it carries within it an important principle, namely that for those localities where the exposure is so good that the surface-wind is near to the geostrophic wind the rate of increase of velocity with height will be slow, whereas in those localities where the surface-wind is a small fraction of the geostrophic wind the rate of increase of the wind from the surface upwards will be rapid. Thus at a station at low level on the coast the wind should increase rapidly from the surface value and attain the geostrophic value at a lower level above the surface than at a high inland station where the rate of approach to the geostrophic wind from the surface would be much lower. In any case, of course, the curve of approach from the surface value of the wind to the geostrophic value is not a straight line but the differences in the average slope of the line for different kinds of exposure are worth consideration and require only three points to be identified, one for the anemometer reading, one for a reading above the anemometer, and the third for the geostrophic wind.

Some observations not yet published with pilot-balloons at Scilly by C. J. P. Cave and J. S. Dines in 1911 show that at Scilly the approach to the geostrophic wind is much more rapid than at Ditcham Park.

Nothing has yet been said about the cases, which will be noticed in any collection of soundings with pilot-balloons[2], in which the velocity of the wind decreases with the height from the surface or from a level not far from it. They are not likely to occur when the surface-wind is in normal relation with the distribution of pressure, but whenever the surface-wind is to be classed as katabatic or anabatic, controlled by gravitational effects at the surface rather than by the general distribution of pressure, the surface-wind may have no relation to the gradient. Such conditions are most likely to occur when the distribution of pressure is very uniform and there is no general gradient to exercise control over the surface-air, and it is to be expected that in these circumstances wind will diminish with elevation and perhaps be reversed at no great height.

On the occasion of a kite competition at Worthing[3] held by the Aeronautical Society of Great Britain on a threatening day in July 1903 the distri-

[1] *First Report Advisory Committee for Aeronautics*, Reports and Memoranda, No. 9.

[2] An instructive collection of diagrams representing a series of soundings with pilot-balloons is given in C. J. P. Cave's *Structure of the Atmosphere in Clear Weather* (Cambridge University Press, 1912).

[3] *The Aeronautical Journal*, January, 1904.

bution of pressure afforded very little prospect of wind sufficient to raise kites, but fortunately for the competition a sea-breeze set in which gave a surface-wind sufficient to get the kites off the ground. In ordinary circumstances, once off the ground, the kite's future is assured because the wind increases aloft, but on this occasion when a certain level was reached the kites lay on the surface-current as if it had been a cushion and travelled along it without making elevation and on that account a very limited height of ascent was possible, hardly exceeding 1800 ft.

It is a well-known experience of meteorologists that winds are sometimes reversed aloft even when they are true to the surface-gradient, and in particular Easterly or North Easterly winds (sometimes also Southerly or Northerly winds, though seldom or never Westerly ones) fall off in the upper air and are replaced by winds from an opposite direction or nearly so. The reversal in these cases however seldom takes place below the level of 500 metres which has been taken as the limit of the surface layers for the purposes of this chapter. It is quite possible that the influence of the surface may extend beyond the level of 500 metres but it is less dominant there.

Reversals at higher levels are controlled by the régime of temperature and their consideration belongs to chap. VII.

The application of the theory of eddy-motion in the atmosphere to the explanation of the variation of wind with height

It will be apparent from what precedes that, even if the exceptional occasions such as those of katabatic winds are left out of account, the variation of wind-velocity with height requires for its representation a whole series of curves all of which, except that representing the case of very light winds, have the same general character and may perhaps belong to the same family. Such a family of curves might be obtained, for example, from an empirical formula for the variation of wind with height such as that of Archibald or of Hellmann by assigning different values to a constant according to the strength of the wind at a standard height, the time of day at different seasons of the year and so on. In any case it is idle to suppose that any empirical formula can hold for winds of all directions at all heights because the régime of winds may change entirely at levels where the distribution of pressure is different from that at the surface. We will, therefore, in this chapter limit our view to the winds which belong to the system indicated by the distribution of pressure at the surface. The lowest kilometre may be taken as a rough indication of the thickness of the layer though the selection of that limit is at present arbitrary.

A rational basis of explanation of the variation of the velocity and direction of wind with height in the lowest layers is to be found in G. I. Taylor's theory of eddy-motion in the atmosphere which accounts for the ascertained facts as dependent upon the turbulence of the lowest layers. The development of the theory is given in a paper to which reference has already been made in the

first chapter[1]. It is based upon a previous paper on eddy-motion[2], the application of which is used in the discussion of the conditions of formation of fog on the banks of Newfoundland in the report of the work carried out by the s.s. *Scotia*, 1913[3]. It is not too much to say that these contributions by their method of treatment of eddy-motion have opened up the field of meteorological investigation in a very remarkable manner. They enable us to obtain a clear insight into the complicated phenomena of the lower layers of the atmosphere, which have not yielded to the ordinary procedure by the method of means.

The fundamental conception is concerned with the distribution of potential temperature in the vertical. In consequence of the turbulence or eddy-motion of the atmosphere heat diffuses downward towards a cold surface according to a law similar to the ordinary law of diffusion of heat in a solid but with a greatly increased coefficient. It may be recalled that the fundamental equation for the diffusion of heat in a solid[4], viz.

$$\frac{d\theta}{dt} = \kappa \frac{d^2\theta}{dz^2},$$

is the expression of the fact that the rate at which temperature is communicated to the solid at a particular point is proportional to the change in the gradient of temperature at that point. That equation when applied to find the distribution of temperature in the atmosphere, on account of the difference of potential temperature between an undisturbed upper layer and a cold surface, similarly expresses the law that the rate at which potential temperature is increased at any point is proportional to the rate of change in the vertical of the variation of potential temperature with height; but when turbulence exists the constant of the proportion is many times that which would be appropriate if the air were solid or restrained in some other way from using its mobility to help towards equalising its potential temperature.

The direct expression for the variation of the potential temperature θ, of unit volume of the air, in terms of the variation of height z, and time t, on the hypothesis set out, is the equation

$$\rho\sigma\delta\theta/\delta t = \frac{\delta}{\delta z}(\kappa\rho\sigma\delta\theta/\delta z),$$

where ρ is the density and σ the specific heat of the air; κ, the eddy-coëfficient of diffusion, is dependent upon the state of the air as regards turbulence.

This equation is reduced to a more simple form as a first approximation by regarding ρ and κ as independent of the height. The specific heat of air is

[1] 'Phenomena connected with Turbulence in the Lower Atmosphere,' *Proc. Roy. Soc.* A, vol. XCIV, p. 137, 1918.

[2] G. I. Taylor 'On Eddy-Motion in the Atmosphere,' *Phil. Trans.* A, vol. CCXV, p. 1.

[3] 'Ice Observation, Meteorology and Oceanography in the North Atlantic Ocean,' *Report...s.s. Scotia*, 1913, *to the Board of Trade.*

[4] Kelvin, *Encyclopaedia Britannica*, Ninth edition, Art. 'Heat.'

known to be practically constant at all pressures and temperatures. Consequently we get the equation

$$\delta\theta/\delta t = \kappa\delta^2\theta/\delta z^2 \qquad \ldots\ldots(1),$$

of which a solution is given by

$$\theta = Ae^{-bz}\sin(2\pi t/\tau - bz) \qquad \ldots\ldots(2),$$

where $$b^2 = \pi/(\tau\kappa)$$

As an example of the application of the theory we may take from the Report of the voyage of the *Scotia*[1] the account of the distribution of temperature and humidity over the sea as disclosed by a sounding with a captive balloon at 7 p.m on August 4, 1913, represented by the curves in fig. 6, p. 47. Taylor traces the course of the air which formed this current from the shores of Labrador along a devious course over the ocean and attributes the reversed lapse of temperature which reaches from the surface to a height of nearly 400 metres to the loss of heat at the surface through the direct effect of the eddy-motion during three days while the air was passing over water colder than itself. The next section of the curve up to 750 metres shows an approximation to the adiabatic lapse for dry air, the result of the passage of air over warmer water as it came down from North to South between July 30 and August 2, while the top portion of the curve shows the remains of the reversed lapse of temperature formed while the air was passing from the coast of Labrador northward over cold water before July 30; the dryness of the air, at the top, itself suggests its origin from over land. Thus he traces in the details of the shape of the curves representing the present distribution of temperature and humidity, the past history of the air and explains the formation of fog in the lowest layer, about 120 metres thick, as a consequence of the mixing of the cold air of the surface with the warmer upper air caused by the eddy-motion. It should be noticed that the cooling effect of eddy-motion has reached nearly 400 metres but the fog only extends about one-third of the way up the line of reversed lapse of temperature which is continuous without change of slope through the fog and beyond it. Also that some days previously the effect of the cold surface had extended upwards beyond a kilometre.

As to the height to which the effect of the eddy-motion extends Taylor gives a formula $z^2 = 4\kappa/(\rho\sigma)\,t$, where z is the height affected by the eddy-motion when the air has been moving over a surface of lower temperature for a time t, κ being the "eddy-conductivity" or coefficient of turbulence and ρ, σ the density and specific heat of the air. The curves of distribution of temperature deduced from that equation as applied to a current of air with an isentropic lapse line passing over cold water are represented in fig. 2 which is also taken from the Report of the *Scotia*. In the diagram the diagonal line represents the supposed original isentropic lapse of temperature with height. The several curves represent the distribution of temperature in the vertical after certain periods of passage over cooling water. The curve marked 1 shows the

[1] *Loc. cit.* p. 53.

height-temperature curve when the travel has been over water which cools the surface air by 1 a in 30 miles run for a period of $1\frac{3}{8}$ hours. That marked 2 shows the effect of a travel of $5\frac{1}{2}$ hours in the same circumstances. For the other three curves a rate of fall of temperature along the surface of 1 a in 60 miles is assumed and the first of them numbered 3 corresponds with a travel of $5\frac{1}{2}$ hours. For No. 4 22 hours are supposed to have elapsed and for No. 5 87 hours. Though the curves do not show the sharp angles that are drawn on

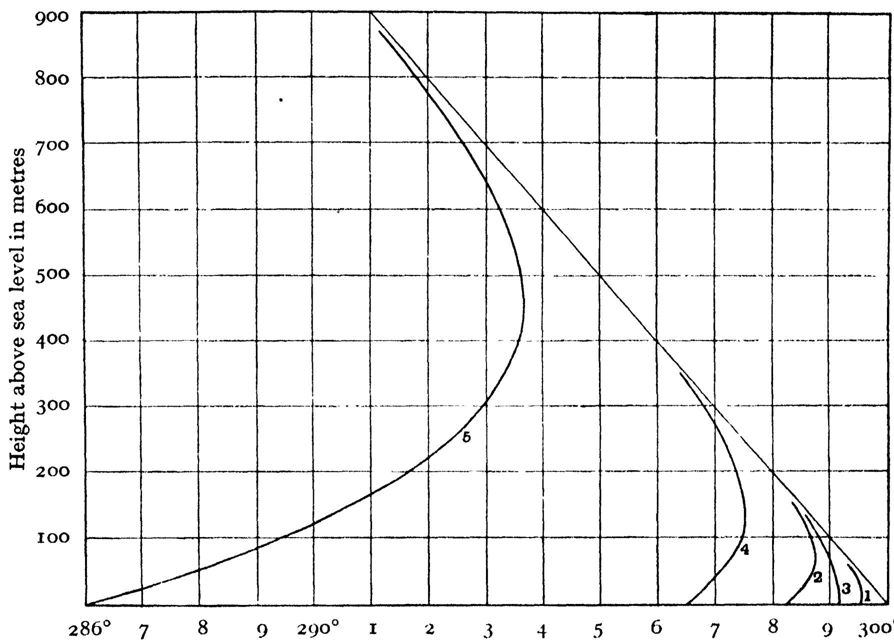


FIG. 2. Curves showing the distribution of temperature set up in an atmosphere originally isentropic, with surface-temperature 300 a, by wind of 4·5 metres per second (10 miles per hour) moving over a track along which the surface-temperature decreases, according to the theory of eddy-motion, with coefficient of eddy-conductivity 3×10^3 C.G.S. units.

Scale of temperature changes.

Curve		hrs.			
1.	Distribution after	$1\frac{3}{8}$	(14 miles fetch)	when temperature decreases	1 a in 30 miles
2.	,, ,,	$5\frac{1}{2}$	(55 ,, ,,)	,, ,, ,,	,,
3.	,, ,,	$5\frac{1}{2}$	(55 ,, ,,)	,, ,, ,,	1 a in 60 ,,
4.	,, ,,	22	(220 ,, ,,)	,, ,, ,,	,, ,,
5.	,, ,,	87	(870 ,, ,,)	,, ,, ,,	,, ,,

p. 47 and other diagrams representing the variation of temperature with height, their shapes are very suggestive of many of the figures that are obtained for the variation of temperature with height from soundings of the upper air. Whenever we have a cold anticyclonic spell of weather in winter, whether there is a fog at the surface or not, the variation of temperature with height is similar to one or other of the five curves of fig. 2, the particular example being

determined by the length of time that the surface has been absorbing heat from the air above it with the aid of the mixing due to eddy-motion.

The value of κ is notably different in different situations, and this we might anticipate from the differences to be expected in the turbulence of the air arising from differences in the nature of the surface which offers resistance to the motion of the air. An order of progressive values of κ may be supposed, beginning with the surface of the sea, or a perfectly smooth surface of land, through the roughness of grass to that of the irregularities of trees or town-buildings the various effects of which are represented in Table II which gives the different ratios of W/G. Some of the values of κ found by Taylor for different situations are as follows:

At sea over the Great Banks (determined from the distribution of temperature) ...	3×10^3 C.G.S. units
Over Salisbury Plain (determined from the distribution of velocity)	5×10^4 C.G.S. units
At the Eiffel Tower (determined from the daily range of temperature at different levels)	
Lowest stage (18 m. to 123 m.) ...	9×10^4 in October to 24×10^4 in March
Mean	15×10^4
Highest stage (197 m. to 302 m.) ...	$1{\cdot}6 \times 10^4$ in February to $30{\cdot}1 \times 10^4$ in July
Mean	11×10^4
Whole range (18 m. to 302 m.) ...	$4{\cdot}3 \times 10^4$ in January to $18{\cdot}3 \times 10^4$ in June
Mean	10×10^4
(Determined by Åkerblom from wind measurements)	$7{\cdot}6 \times 10^4$

In like manner, as the surface may act as a boundary at which heat is absorbed, so it may act as a boundary at which momentum is absorbed; and in that case momentum also diffuses downward from the "undisturbed" current of air in the upper regions to be lost at the surface, and the distribution of velocity with regard to height and time will follow the law of the equation

$$\rho \delta u/\delta t = \kappa \rho \delta^2 u/\delta z^2 \qquad \ldots\ldots(3),$$

where κ has the same value as in the thermal equation because the diffusion both of heat and momentum is governed by the eddies which cause the mixing of the layers.

"Roughly κ may be taken as $\frac{1}{2}wd$ where w represents the mean vertical component of velocity due to the turbulence and d represents roughly the mean vertical distance through which any portion of the atmosphere is raised or lowered while it forms part of an eddy till the time when it breaks off from it and mixes with the surroundings. This may be taken to be roughly equal to the diameter of a circular eddy."

For the steady state under these conditions, assuming that the system has been established long enough for the initial conditions to have died away, Taylor has obtained a formula for the relation of the surface-wind to the

undisturbed wind (taken as equivalent to the geostrophic wind) in terms of the angle α of deviation between the geostrophic wind G and the surface-wind W, which takes the form

$$W/G = \cos\alpha - \sin\alpha \qquad \text{......(4).}$$

The equation is tested by comparison of the calculated results with some observations of the surface velocity and its angular deviation from the gradient, by G. M. B. Dobson[1] over a very suitable exposure on Salisbury Plain; the comparison is as follows:

	Light winds	Moderate winds	Strong winds
Observed value of W/G ...	·72	·65	·61
α observed	13°	21½°	20°
α calculated	14°	18°	20°

The closeness of the agreement is remarkable and incidentally we may note that in this case also the lighter winds show closer agreement with the gradient both as regards direction and velocity. The comparison is, moreover, supported by the examination of a conclusion arrived at by Dobson from his observations, namely that the geostrophic velocity is attained at a height considerably below that at which the direction of the geostrophic wind is reached. The difference is usually as much as that between 300 metres for the velocity, and 800 metres for the direction, and this is shown to be a direct consequence of the theory.

From the application of this theory Taylor arrives at the further conclusion that for a given geostrophic wind G, which is reduced by surface friction to such an extent that the surface-wind is inclined at an angle α to the undisturbed wind, the rate of loss of momentum to the surface, that is the force of surface friction F, is

$$2\kappa\rho G \sin\alpha/B \qquad \text{......(5),}$$

where B is equal to $\sqrt{\omega \sin\phi/\kappa}$; ω is the angular velocity of the earth's rotation and ϕ is the latitude. And in another paper[2] he obtains for the frictional force of air over the grassy land of Salisbury Plain the value

$$F = 0{\cdot}0023\rho W^2,$$

whence

$$0{\cdot}0023 W^2 = 2\kappa G \sin\alpha/B.$$

Substituting numerical values of ω and ϕ, and remembering that W/G is equal to $\cos\alpha - \sin\alpha$, we get

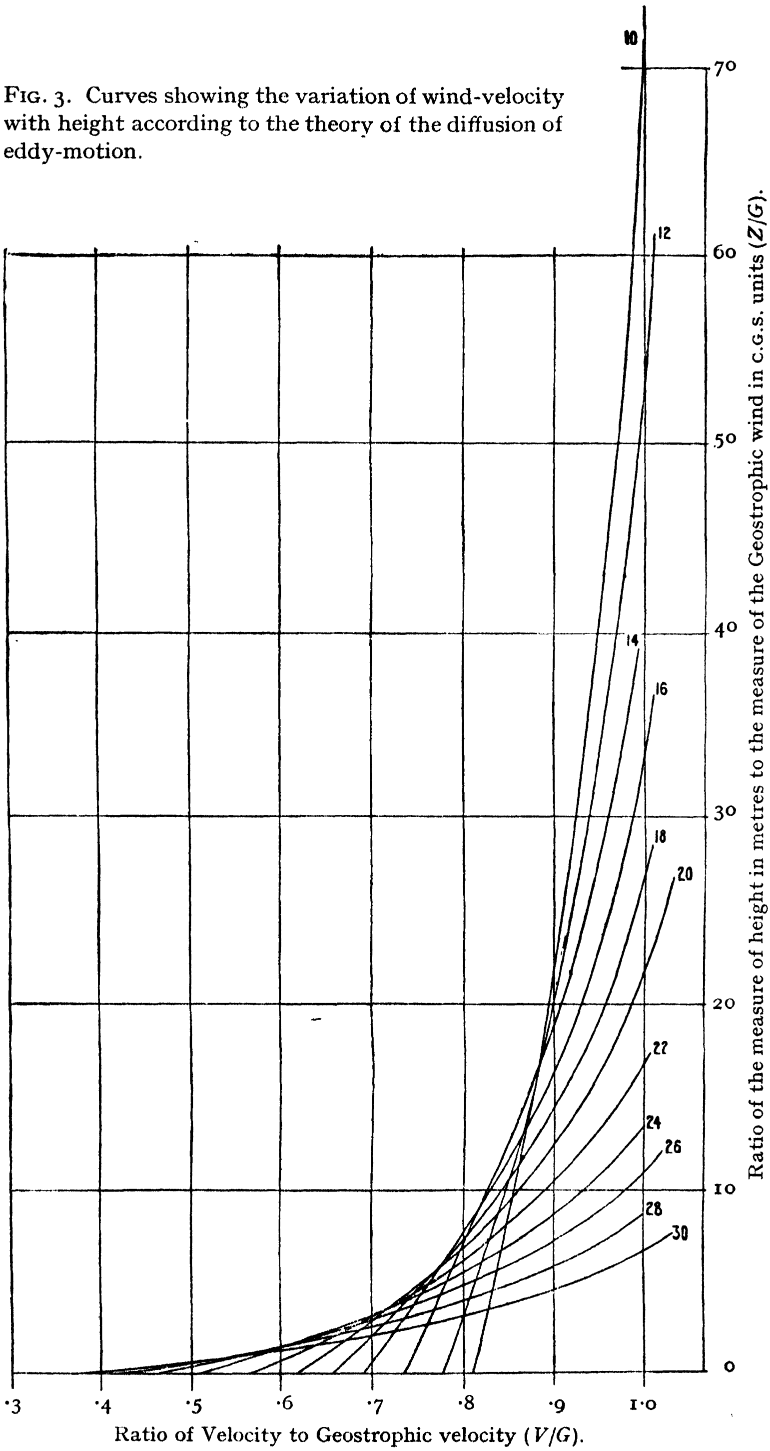
$$\frac{1}{BG} = \frac{20{\cdot}4}{\sin\alpha}(\cos\alpha - \sin\alpha)^2.$$

In deducing equation (4) the following equation was obtained[3] for the component u of wind-velocity along the isobar:

$$u = G - A_2 e^{-Bz}\cos Bz + A_4 e^{-Bz}\sin Bz \qquad \text{......(6),}$$

[1] *Q. J. Roy. Met. Soc.*, vol. XL, p. 123, 1914.
[2] *Proc. Roy. Soc.* A, vol. XCII, p. 198, 1916.
[3] 'On Eddy-Motion in the Atmosphere,' *loc. cit.* p. 15.

FIG. 3. Curves showing the variation of wind-velocity with height according to the theory of the diffusion of eddy-motion.



The figures marked against the curves in the body of the diagram give the deviation α of the wind at the surface from the undisturbed or geostrophic wind in the "free air." It should be noted that the maximum value of α is 45° when, according to the formula, the surface-wind becomes zero.

where $$A_2 = G \tan\alpha\,(1 + \tan\alpha)/(1 + \tan^2\alpha),$$
and $$A_4 = G \tan\alpha\,(1 - \tan\alpha)/(1 + \tan^2\alpha),$$
from which we obtain
$$(G - u)/G = e^{-Bz} \sin\alpha\,\{\cos(Bz - \alpha) - \sin(Bz - \alpha)\},$$
as an equation for the relation between velocity and height. Since
$$\kappa = \omega \sin\phi/B^2,$$
the relation will be different for different values of κ, and if κ is subject to diurnal variation there will be a corresponding diurnal variation in the curves which represent the relation of velocity and height.

The corresponding values of α, B, and κ for values of G to be taken at will, are given in the following table.

TABLE III.

α	BG	κ/G^2
°	C.G.S. units	C.G.S. units
4	·0040	3·54
6	·0065	1·35
8	·0094	0·635
10	·0129	0·338
12	·0171	0·192
14	·0223	0·116
16	·0286	0·069
18	·0366	0·042
20	·0456	0·027
22	·0599	0·0156
24	·0775	0·0094
26	·101	0·0055
28	·135	0·0031
30	·181	0·0017
32	·270	0·00085
34	·384	0·00038
36	·588	0·00016

With these values of the constants Taylor has constructed the curves which are represented in fig. 3, the shapes of the curves being determined by selected values for α at the surface marked against them. The abscissae are the ratios of the wind-velocity to the geostrophic wind and the ordinates are the ratios of the numerics of the height and the geostrophic wind, so that, for example, when the geostrophic wind is 10 m/s the figures at the side will represent heights in dekametres, and those at the base velocities in dekametres per second.

It is interesting to note the difference in the curves which would be suitable for representing the variation of wind with height under different conditions as to turbulence. If we take from p. 36 the three values of the coefficient κ, 3×10^3, 5×10^4 and 10×10^4 appropriate for the sea, for Salisbury Plain and for Paris respectively, we see that to give a value of α equal to 20° (requiring a

quotient 0·027 for κ/G^2) the geostrophic winds would have to be about 3 m/s, 12 m/s and 18 m/s respectively. Hence different curves are appropriate for the same geostrophic wind if on account of the circumstances of the site, the time of day or the season of the year the coefficient κ which defines the turbulence is different, a conclusion which we have already seen to be in accord with experience.

This adjustment to the different circumstances of the site or time is a strong point in favour of the theory as a dynamical explanation of the variation of wind with height and the evidence is strengthened by the further application of the theory, in the paper from which we are now quoting, to calculate, from a transformation of the curves which we have reproduced, the heights at which the day maximum of wind-velocity with which we are familiar at the surface gives place to the day minimum which was represented in the results for the Eiffel Tower and is characteristic of all mountain stations and, as we have already seen, is disclosed in the results obtained with pilot-balloons at South Farnborough. It appears as a result of these calculations that a variation in κ by an amount which fits in well with all the other known data concerning the turbulent motion of the air near the ground is sufficient to explain both quantitatively and qualitatively all the facts concerning the daily variation of wind-velocity at different heights above the ground which are brought to light by Hellmann's observations.

It will be clear to the reader from the imperfect sketch which we have been able to give of the theory of eddy-motion that κ is a new meteorological quantity of great importance. Its influence is chiefly at the surface but on occasions it extends to heights above a kilometre. We shall see in the next chapter some indirect results that may follow from its influence upon the atmosphere.

Note. Captain D. Brunt has pointed out that equation (6) of p. 37, with the equation for the corresponding component v at right angles to the isobar,

$$v = -A_2 e^{-Bz}.\sin Bz - A_1 e^{-Bz}\cos Bz,$$

leading to

$$v = Ge^{-Bz}\sin\alpha\,\{\cos(Bz-\alpha) + \sin(Bz-\alpha)\},$$

implies that the point of the vector representing the velocity at successive heights traces out an equiangular spiral.

CHAPTER V

Turbulence in relation to gustiness and cloud sheets

THE theory which has been adduced in explanation of the various contingencies in the relation of the surface-wind to the gradient receives strong support of an incidental character from what we have learned in recent years about the gustiness of ordinary winds. When the tube-anemometer, devised by W. H. Dines in 1890, was set in operation the wind was seen from the records to consist of a series of rapid alternations of velocity, and when a direction recorder was subsequently added the alternations in velocity were found to be accompanied by corresponding alternations in direction. The

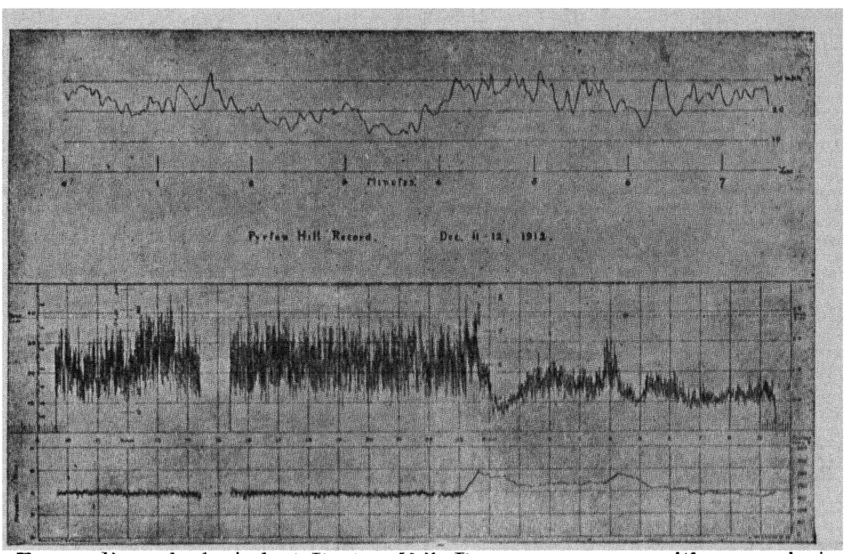


FIG. 1. Record of wind at Pyrton Hill, Dec. 11–12, 1912, with an analysis of the gusts during a quick run for seven minutes between 14 h. 30 m. and 15 h. 20 m.

number in a given time and the range of consecutive fluctuations which make up these alternations are quite irregular. We have become accustomed to refer to them as the "gustiness" of the wind. The word is not very appropriate because besides these rapid fluctuations of velocity there are other marked increases in the velocity of many winds lasting for some minutes, which we call squalls, and the transition between the normal fluctuations in the wind and the occasional squall is not well expressed by it.

The nature of ordinary gustiness will be best understood by the study of the illustrations of the records obtained from an anemometer with its vane

98 ft. above the ground at Pyrton Hill by J. S. Dines[1] and reproduced here. They show the separate records of velocity and direction for two occasions with the ordinary time-scale (2 cm. equivalent to 5 hrs. in the reproduction) and above the regular record in each case is an inset representing the velocity during "quick runs" on the time-scale of 10 cm. to 8 minutes.

We may note that in the quick runs the trace of the velocity is a very irregular line and on the closer time-scale the trace appears as an irregular ribbon; the width of this ribbon represents the range of the gusts of which the wind is composed. It will be noticed that speaking in general terms the width of the ribbon varies with the mean velocity of the wind and is roughly speaking proportional to it. It may thus be taken as a measure of the gustiness of the wind which passes the anemometer, and as a numerical measure applicable to winds of different strengths we may use the ratio of the width of the

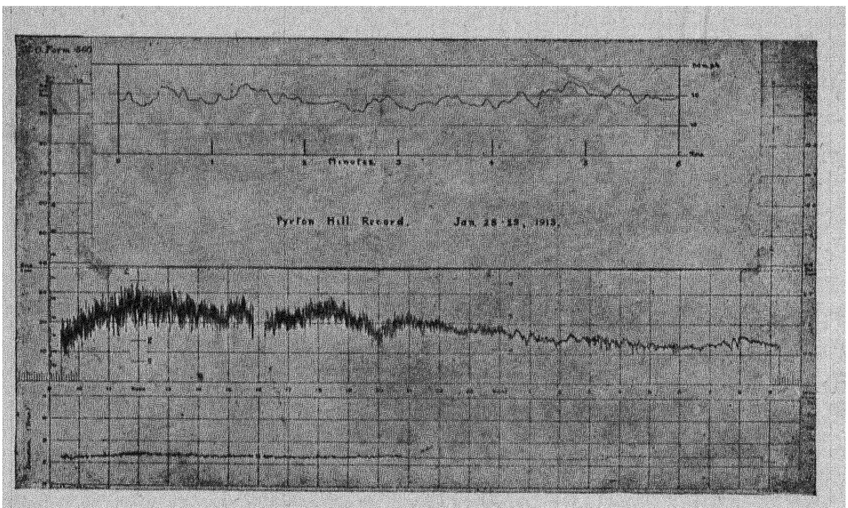


FIG. 2. Record of wind at Pyrton Hill, Jan. 28–29, 1913, with an analysis of the gusts for six minutes at 16 h.

ribbon in a short interval of time to the mean velocity for that interval. Using this measure we find that the gustiness of winds is different for different situations and is, moreover, different for different orientations in the same situation; in some cases, as, for example, at Shoeburyness, where there is a land-exposure on the one side and a sea-exposure on the other the difference of gustiness for different orientations is very marked.

Judged by this standard we find that the most gusty exposure for the stations with tube-anemometers which report to the Meteorological Office is that of Dr J. E. Crombie of Dyce, near Aberdeen, where the mast of the anemometer projects fifteen feet above surrounding tree tops; the coefficient of gustiness in this case is 1·3.

[1] Advisory Committee for Aeronautics, *Report*, 1913 *Fourth Report on Wind Structure*, fig. 1 and fig. 2 Blocks lent by H.M. Stationery Office

For other sites we have the following:

Marsh Side, Southport	Coefficient of gustiness	·3
St Mary's, Scilly	,, ,, ,,	·5
Shoeburyness, ENE wind ...	,, ,, ,,	·3
,, W wind	,, ,, ,,	·8
Holyhead (Salt Island)	,, ,, ,,	·5
Pendennis Castle (Falmouth), S wind	,, ,, ,,	·25
,, ,, ,, W wind	,, ,, ,,	·5
Aberdeen (Roof of King's College)	,, ,, ,,	1
Alnwick (Roof of Schoolhouse) ...	,, ,, ,,	·8
Richmond (Roof of Kew Observatory)	,, ,, ,,	1

The gustiness which is represented by these figures may be regarded as the direct result of the eddy-motion or turbulence due to the obstacles which are presented to the direct flow of the air by the surface and its irregularities, the eddies being more pronounced the greater the resistance presented by the obstacles. Where the surface is nearly flat, as over the sea or a flat spit of land, the turbulence is less marked, but when the air has to make its way over trees or groups of buildings the eddies are larger and more pronounced and the turbulence produces greater effect on the record of the anemometer.

A useful piece of evidence confirming this view comes from Eskdalemuir where the anemometer is mounted on the central block of the Observatory. It stands between two other buildings on a hill-side which slopes towards the East. Above the central block on the West is the Superintendent's house, below it on the East is the Assistants' house. Nothing particular is remarked about the Easterly winds which pass over the lower structure; but whenever the wind veers from South to West the passage of the wind-shadow of the house at the higher level across the central block is always marked on the record by an increase of the gustiness shown in the traces both of velocity and direction.

J. S. Dines[1] has put together some information about the variation of gustiness with height. He found for two anemometers, one with its vane at 36 ft. and the other at 98 ft., that the gustiness factor of the lower (estimated as the ratio of the range of velocity during an hour to the mean velocity for the hour) was 137 per cent. of that of the higher, and from observations of the variation of the pull of a kite-wire he found the gustiness to vary with height very differently on different occasions with the general average of a factor of gustiness (range of pull on the kite ÷ mean pull) of 2·5 for the step 0 to 500 ft. and 1·5 for the step 500 to 1000 ft. Four ascents with a Westerly wind gave for the lower step irregular values 2·6, 3·7, 3·5 and 1·3 and for a South-Westerly wind 2·0, 4·5, 2·0 and 1·4, while for seven ascents when the wind had an Easterly component the factors were all high but more uniform, namely 2·7, 3·0, 7·3, 2·7, 3·2, 3·5, 2·5. The station is situated on the Western slope of the Chiltern Hills. Easterly winds come over the hills.

[1] Advisory Committee for Aeronautics, *Report*, 1911–12. *Third Report on Wind Structure*, p. 219. 1910–11, *Second Report on Wind Structure*. Reports and Memoranda, No. 36.

These factors of gustiness appear to be directly related to the constant κ of Taylor's formula and the whole question of gustiness seems to be merely a phase of the general question of eddy-motion.

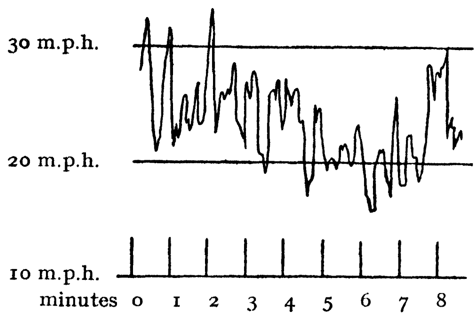


FIG. 3 *a*. Ordinary Gustiness.

An investigation of the details of the motion represented by gustiness has been made by J. S. Dines[1]. He has pointed out the difference between the record of an anemometer with an open time-scale, represented by fig. 3 *a*, and the sudden changes in wind-velocity, represented by the examples of fig. 3 *b*, which last for a minute or more and would be appreciable by an aeroplane as a definite "bump" in consequence of the almost instantaneous transitions between phases of the

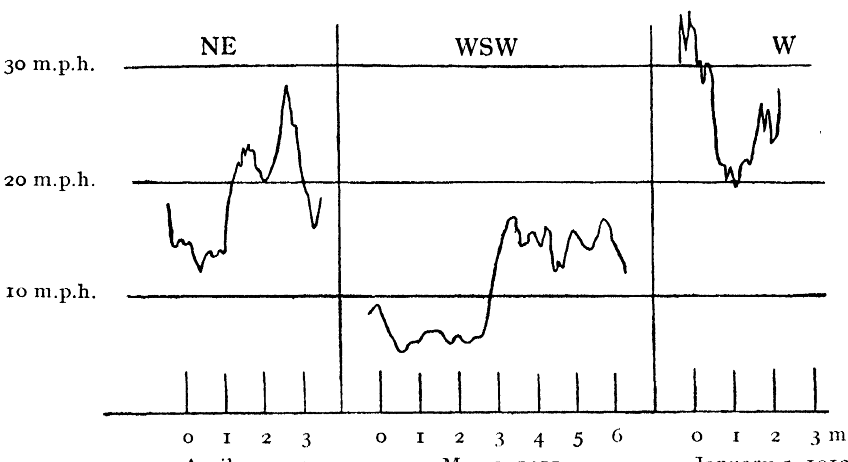


FIG. 3 *b*. Sudden changes of wind-velocity of comparatively long duration.

relative motion which last long enough to alter the lift, whereas the fluctuations of ordinary gustiness are alternations which are complete in a few seconds.

An endeavour made by J. S. Dines[2] at the writer's suggestion to analyse the motion of eddies by arranging an anemometer to give a vector-diagram representing the direction and speed of the wind at each moment by a vector-radius drawn from a centre, has failed to indicate any simplification of the idea of turbulent motion in the layers of air near the ground. The diagram

[1] Advisory Committee for Aeronautics. *Third Report on Wind Structure*, p. 216, 1912.

[2] Advisory Committee for Aeronautics. *Second Report on Wind Structure*, 1911, Plate 5, fig. 6.

(fig. 4 *a*) reproduced here shows the record, lasting one minute, of some exceptionally variable winds from the North East at 36 ft. A corresponding diagram (fig. 4 *b*) with wind of similar range of velocity from the West at 98 ft. shows much less violence of oscillation in direction but it must be noted that the large range of the oscillations shown in the diagram may be due to the momentum of the vane which may carry the writing pen beyond the true position of the wind, and in this connexion it may be remarked that with the

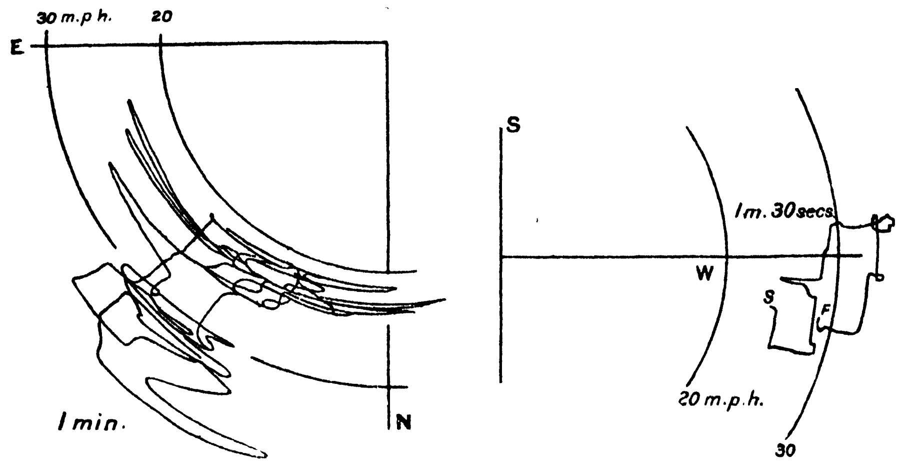


FIG. 4*a*. FIG. 4*b*.

FIG 4. Vector diagrams of variation of velocity of wind at Pyrton Hill.

view of reducing oscillations of this character a new form of vane has been designed and is now in operation on Pyestock Chimney, South Farnborough.

J. S. Dines has also taken records of the variations in altitude of a small balloon tethered by a thread 100 ft. long to the top of the anemometer pole

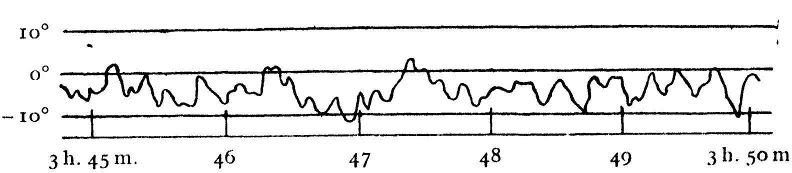


FIG. 5. Theodolite-record of altitude of a tethered balloon.

98 ft. high at Pyrton Hill. One of these records for January 19, 1912, is represented in fig. 5, and is reproduced in order to show that the vertical fluctuations of short period in an air-current are of the same kind as the fluctuations in the velocity and direction of the horizontal motion as represented in the trace of an anemograph. It follows that the effect of the imperfect eddies due to the turbulence of the flowing air is to produce the same kind of alterations in the horizontal and vertical directions; and, consequently, the change in direction, the change in the horizontal speed of the wind and the superposed

vertical fluctuations may all three be regarded as aspects of the same physical or dynamical process.

G. I. Taylor has carried the theory of eddy-motion so far as to show that there is a direct numerical relation between the mean variation in the horizontal direction of the wind and that in its speed[1]. He has, moreover, obtained evidence to show that the effect of the turbulence spreads equally in all directions round any selected point.

The Relation of Turbulence to the Formation of Clouds

The first application of the theory of eddy-motion in the atmosphere was the explanation of the formation of fog over the banks of Newfoundland[2]. It was shown by G. I. Taylor that by taking account of the eddy-diffusion between a current of air in the upper regions and a surface of cold water the distribution of temperature in the lowest layers of the atmosphere determined by soundings with kites from the deck of s.s. *Scotia* could be explained. A specimen of this distribution of temperature observed during the occurrence of fog is shown in fig. 6. It is quite typical of the whole series of observations and the general application of the type is borne out by many other observations of the temperature of the air in fog which show that fog is always associated with an inversion of the lapse of temperature with height[3]. The coldest stratum is at the ground and the temperature gradually increases upwards within the fog and for some additional height beyond it. The cloud of fog is the immediate effect of the mechanical convection of cold caused by the mixing of consecutive strata in the turbulence of the eddy-motion and in spite of the fact that the coldness of the lowest layers makes for stability and restrains convection. The formation of the cloud extends as far upward as the reduction of temperature due to the mechanical process of the eddies extends sufficiently to produce a mixture which has a temperature below its dew point. When condensation has begun the further effect of the turbulence is to mix the fog-laden layers as well as to extend their upward boundary so that the water condensed at any particular level has not all to be borne by the air at that level, and the thickness of the cloud is more uniform than the ascertained variation of temperature would lead us to expect.

We must therefore picture to ourselves a set of rolling eddies producing cloud by mixture beginning at the surface and gradually extending upwards as the current flows on. The operation is very persistent if the conditions are maintained, yet it is very self-contained because the coldness of the surface layer always tends towards stability and therefore towards limiting the operation to those layers which are directly affected by the eddies set up at the

[1] *Report Advisory Committee for Aeronautics*, Reports and Memoranda (New Series), No. 345 (unpublished).

[2] *Report of s.s. Scotia to Board of Trade*, 1914.

[3] The reader may refer to the results of observations with kites (particularly those at Brighton by S. H. R. Salmon) published in the *Weekly Weather Report*, 1906 to 1911, and subsequently in the *Geophysical Journal*, M. O. Publication, No. 209 d, etc.

surface and gradually developed in higher levels, but only by gradual incorporation of the next higher layer with those already affected.

This view of the formation of fog being accepted we have next to note that many, if not all, of the varieties of stratus cloud are also marked by an inversion

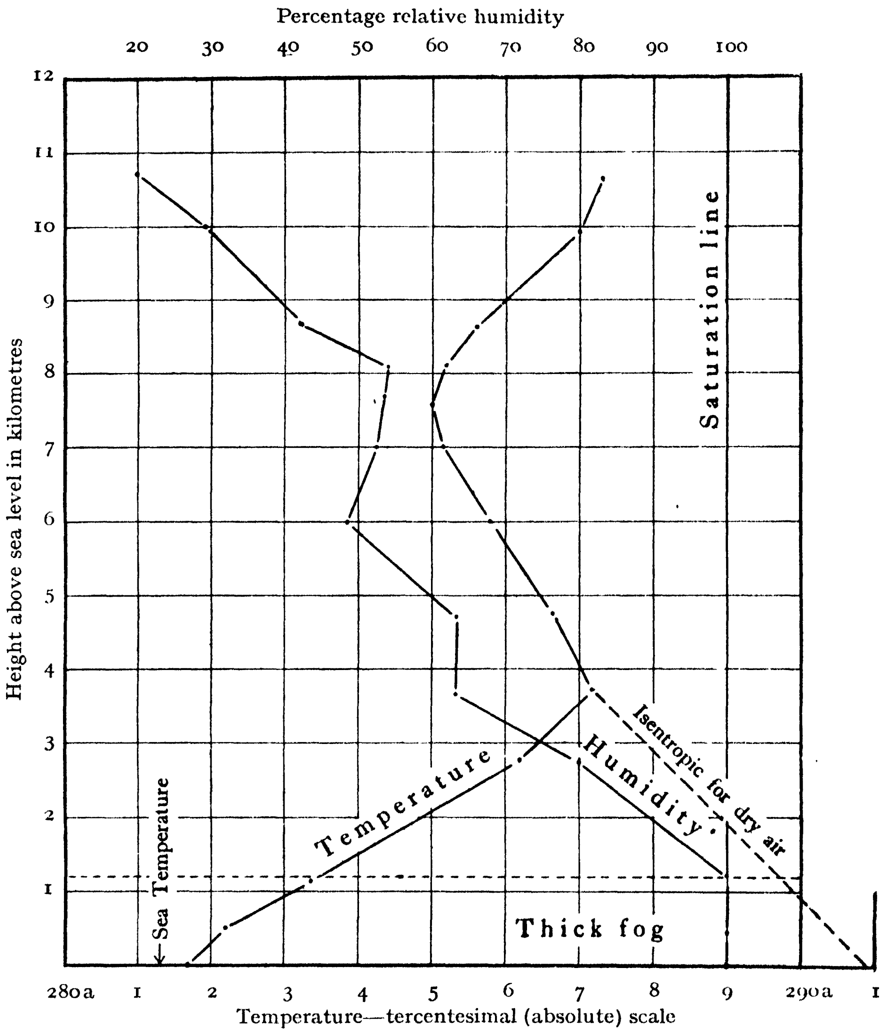


FIG. 6. Captive balloon ascent from s.s. *Scotia*, East of Newfoundland, August 4, 1913, 19 h. Wind 5 miles per hour (2·2 m/s) at all heights from SE¼S (140°).

of lapse of temperature in their mass, extending beyond their upper limits. This has been noted on many occasions during kite ascents. Not all clouds have that characteristic but only clouds of certain types such as stratus and strato-cumulus. Cumulus clouds have no inversion of temperature at their tops.

In explanation of this peculiar separation of clouds into two different classes let us consider what would happen if a current, such as that which produces fog gradually extending upwards from a cold surface, passed over a surface which was not colder than but as warm as or warmer than the lowest layer of the moving air. The turbulence due to the eddy-motion would of course exist in that case also, and its coefficient of diffusion upward would be greater than that for a cold surface. The effect of the turbulence would therefore be to cause the layers affected by it to approach the isentropic condition for dry air, that is, to make a greater approach to uniformity of distribution of potential temperature by the gradual process of mixing or churning in the layers affected. This implies a sacrifice of temperature in the upper layers to the advantage of the lower layers. Hence would arise a reduction of temperature of the upper boundary of the stratum affected, as compared with the layer just above it, and consequently just beyond the influence of the turbulence. In other words an inversion would be formed marking the boundary of the stratum affected by turbulence just as it does in the case of a stratum in which fog is formed; in which, indeed, the process is exactly similar in this respect, viz., that the change of temperature from the surface upwards is less abrupt from cold to warm and therefore is nearer to the isentropic lapse than if there had been no mixing of the lower layers.

But it is clear that if the air in eddy-motion at any point can be represented as having a vertical component for part of its course there will come a time when the vertical elevation will reduce the temperature of the air below its dew point; cloud will therefore form and the top of the stratum affected by the turbulence will be marked by a layer of clouds or cloudlets, always being formed so long as the eddies persist, travelling with the wind while they are in existence and always being reformed with sufficient regularity to give the idea of a permanent drifting layer. Such clouds are not likely to develop into rain as a general rule because above them by the process of their formation is a layer of inverted lapse which is a guarantee of stability, but so long as the surface conditions and the current of air above the surface are maintained so long will the formation of cloud take place. And it will give a very level under surface because the condensation will always take place under exactly similar conditions of temperature which range themselves in horizontal layers; but different samples of air may well have different amounts of moisture and clouds may therefore form in irregular patches or in rolls. And the difference in the amount of condensation may occasion local differences in the thickness of the layer of cloud. The appearance of a layer of strato-cumulus cloud from below is familiar enough. The development of the art of flying and of photography in connexion therewith has placed a new method of observation at our disposal. We reproduce, with remarks based upon the original notes, a number of examples of photographs of clouds from above which have been supplied by Captain C. K. M. Douglas, R.A.F., through the courtesy of the commandant of the Meteorological Section of the Royal Engineers in France.

EDDY CLOUDS: PLATES I, II AND III

PHOTOGRAPHS OF CLOUD-SHEETS FROM AEROPLANES

WITH REMARKS BASED UPON NOTES CONTRIBUTED BY

CAPTAIN C. K. M. DOUGLAS, R.A.F.

Nos. 1, 2, 3, 4, *Sheets of strato-cumulus cloud in ripples, waves or rolls.*

Examples 1, 2 and 4 were taken on the same day, August 15, 1918; the first, at 1700 feet, in the early morning about 7 h, and the other two in the evening at 18 h when the cloud-sheet of the morning had worked upwards and developed a much more turbulent appearance. The tops of the rolls were then at 5000 feet. The clouds were formed in a light northerly wind on the eastern side of an area of high pressure lying over the English Channel.

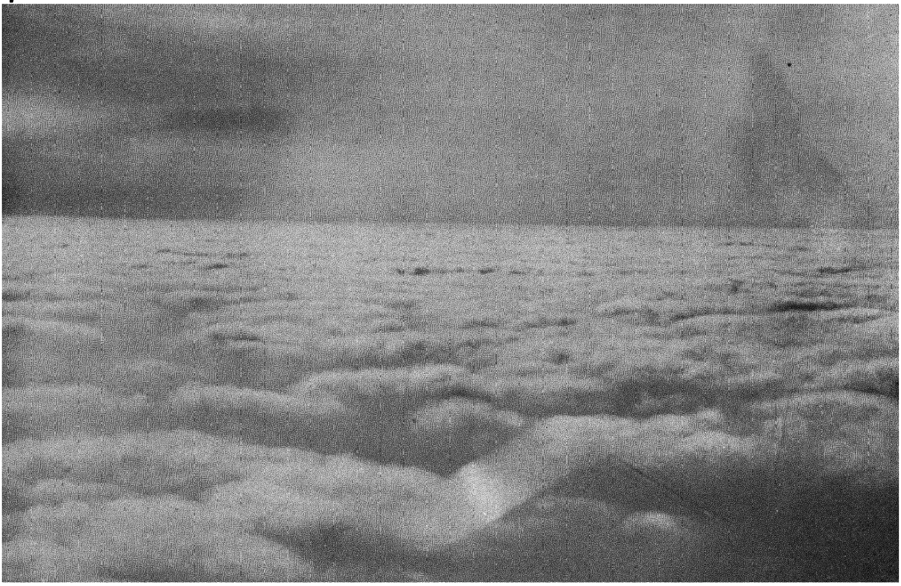
The third example shows a cloud-sheet with tops at about 4000 feet formed in a fresh westerly wind at 7 h. 30 m. on August 17, 1918.

"These clouds are accompanied by eddy-motion within and below them which keeps up a supply of water-vapour from below. It is the expansion and cooling of this water-vapour as the air carrying it gets diffused up by eddies that causes the clouds. Similar cloud-sheets are common at all heights up to 20,000 ft. The turbulence reduces the temperature at the cloud-level and there is often a rise of temperature above it." It was only 1° F. in the first example but in the later examples of the same day when the cloud was several thousand feet higher it had increased to 8° F.

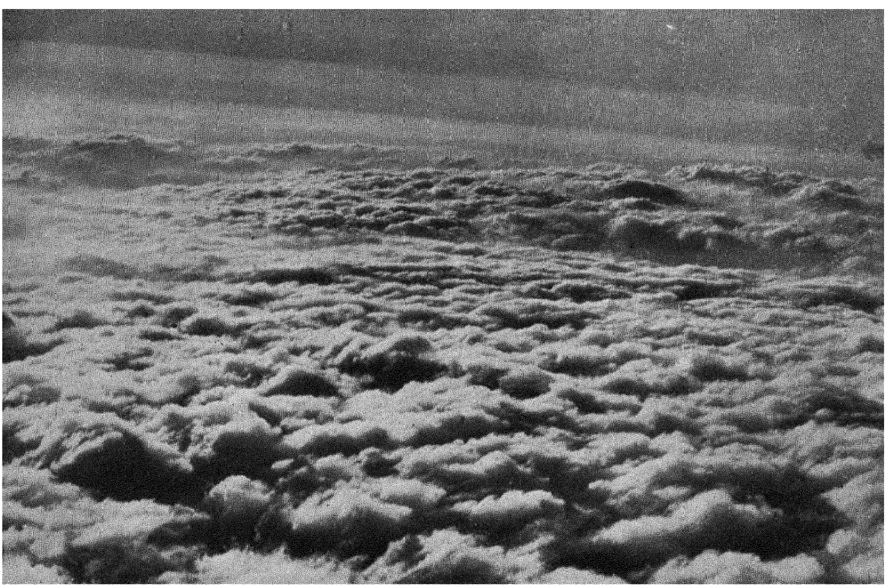
Nos. 5 and 6. *Low Clouds of Lenticular Type.*

"These clouds are of interest as they represent a type rather similar to those shown in Plate XIV of the series recently published by the Meteorological Office[1]. They were however at a much lower altitude, about 2500 to 3000 feet at their upper surface. They were accompanied by very little turbulence and occurred in a stable layer, the temperature being 53° F. at 2000 feet and 50° F at 4000 feet." The normal lapse of temperature between these levels is between 5° and 6° F. The peculiarity of the lenticular clouds which have the smooth, gently rounded form so well imitated by the long wreaths or "sastrugi" of examples 5 and 6 is that they seem to be the loci where cloudlets are persistently formed and move independently of the general motion of the cloud-bank. The banks in this case lay in North and South lines and moved from SSW. Comparing their forms with the diagrams of wave-motion in figure 1 of chapter IX it is impossible to resist the suggestion that these long banks may represent waves, which are stationary or nearly so, across the current of air which forms the wind. On this occasion there was a wind at 7000 ft. from WSW, while the surface-wind was from the South. Northern France was under the northern margin of an extended anticyclone and there was a stationary "low" centred off the Hebrides. [7 h. 30 m. Aug. 8, 1918.]

[1] *Cloud Forms according to the International Classification*, M. O. Publication, No. 233, 1918.

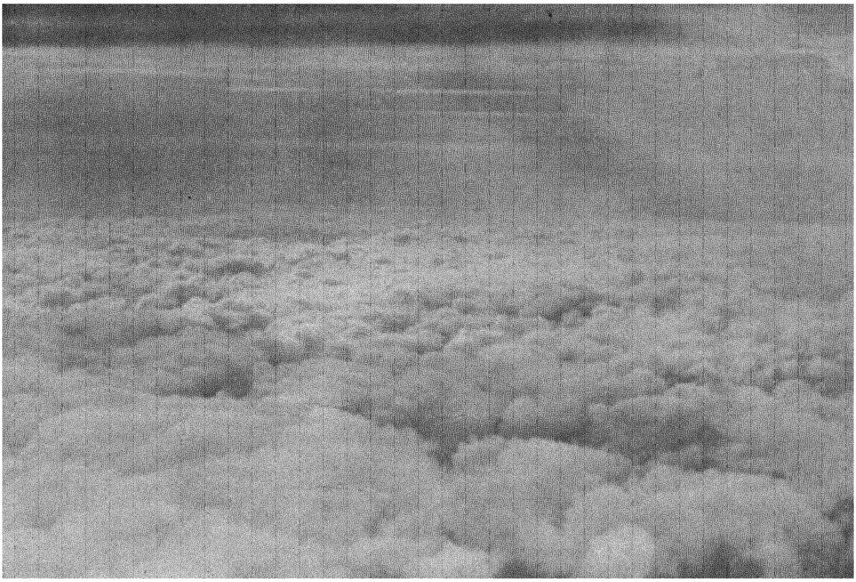


1. Flat sheet of cloud at about 1700 feet, in rolls or waves of strato-cumulus advancing towards the observer. August 15, 1918, about 7 h.

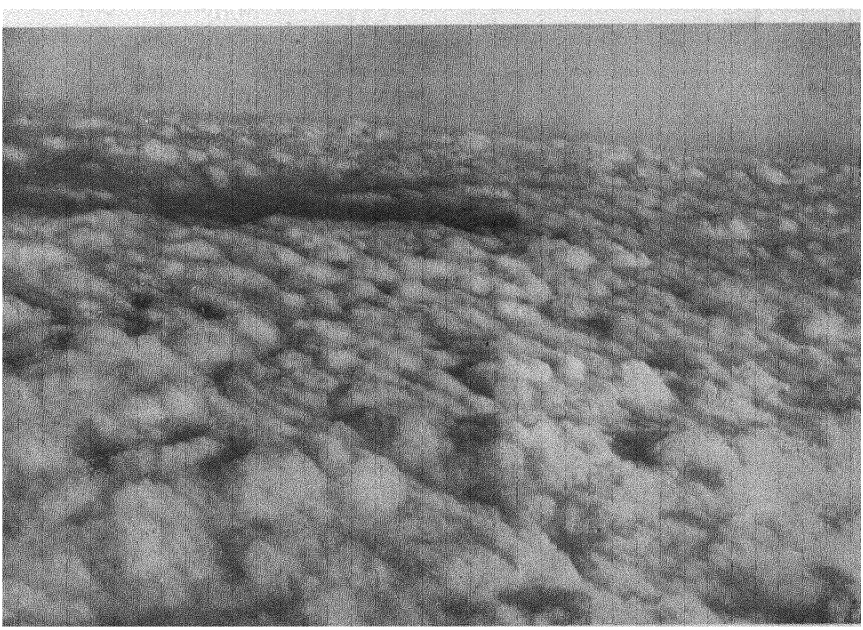


2. The same layer of clouds as No. 1 at a greater height in the evening of the same day. Strato-cumulus in rolls and hummocks with tops at 5000 ft. advancing from the North (obliquely from the right towards the observer facing North-west) with sunlight from the West. The sun is out of the picture on the observer's left. August 15, 1918, 18 h.

Note the clearer belt beyond the strato-cumulus, over the English Channel, and the bank of clouds on the horizon over England.

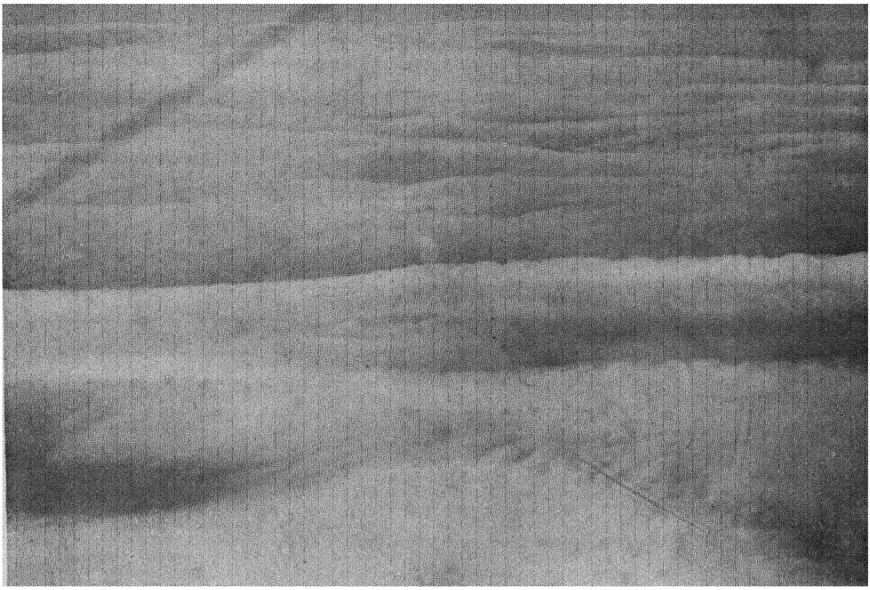


3. **Strato-cumulus** clouds about **4000 feet** with high clouds in another layer above, **probably alto-stratus** which is generally **at 10,000** feet or higher. August 17, 1918, 7 h 30 m.



4. **Another** view of the cloud-sheet represented in No. 2 on the opposite page **looking East with the sun** behind the **observer**. The lines of cloud are moving obliquely away **from the observer** from the **foreground** on the left to the background on the right. **August 15, 1918, 18 h.**

Note the **more turbulent** appearance of the evening clouds represented in Nos. 2 and 4 as **compared with the morning** clouds represented in Nos. 1 and 3.



5. A vast field of long strips or bands of cloud nearly parallel, with smooth and lightly rounded upper surfaces in the form of "hogs-backs," like wreaths of drifted snow (sastrugi).

6. Another part of the cloud-sheet, No. 5, viewed to NE against the light showing an eruption of cumulus heads in the middle distance. Overhead is another cloud sheet at 7500 ft, of which the margin is seen at the top of the picture. August 8, 1918, 7 h 30 m
The bands of cloud lay roughly North and South. The wind at their level was from SSW. The form suggests wave-motion across the bands from W to E

There are a number of well-known phenomena which may be accounted for on this hypothesis of cloud marking the boundary of the stratum affected by the turbulence of the surface. There is first of all the well-known circumstance that the sky over our Islands may be covered by a layer of cloud the whole day through without allowing a break for the sun on the one hand or developing into rain on the other. The adjustment which is necessary to cause condensation without rain is a very delicate one. It could hardly be attempted in a physical laboratory by any simple process of reduction of air pressure; but if arrangement could be made for a certain limited amount of rarefaction to be superposed upon a gradual diminution of temperature with height the necessary adjustment might be made and such an arrangement seems to be secured by the effect of eddy-motion upon a flowing current when the surface is as warm as, or warmer than the air above it. The explanation is even more *à propos* if we consider that layers of cloud such as are here spoken of have a marked tendency to disappear over land stations towards or after sunset when the level reached by the turbulent motion becomes lower. We have seen that the height to which turbulence extends has a marked diurnal variation, being increased by the accumulated warmth of the day. D. Brunt[1] has pointed out that there is a diurnal variation of cloud at Richmond (Kew Observatory), which is a very good indicator of the diurnal variation of the thickness of the layer affected by turbulence.

Clouds of this character are often formed in Easterly winds, which at the surface are dry winds. They are indeed so characteristic of anticyclonic weather in winter, that they have received from W. H. Dines the name of anticyclonic gloom. It should be noted that these clouds are to be found in the current which brings the surface-wind. They do not belong, as one is apt to think, to the transition between the surface-wind and upper winds in the opposite direction coming from a warmer quarter with a larger supply of water, or to any other process of mixing of currents from different sources in the upper air; they are developed in the body of the surface-current itself[2].

Another example which has frequently been noticed in the course of the last four years is the heavy cloud in winds from the North which have a long

[1] *M. O. Professional Notes*, No. 1.

[2] In a letter recently received W. H. Dines has expressed the opinion that loss of heat by radiation is the most probable cause of the cloud in the still air of a winter anticyclone and other clouds of stratus-type. The suggestion deserves more careful consideration than is possible at this stage. The relation between the ultimate effects of radiation and the formation of cloud is a very complicated thermodynamical process. The immediate effect is thermal: the next step is the dynamical process of the adjustment of level according to the entropy or potential temperature. The final result of local loss of heat by radiation may be the "warming" of the air which has lost its heat. See Shaw, 'La Lune mange les Nuages,' *Q. J. Roy. Met. Soc.*, vol. XXVIII, p. 95, 1902, reproduced in *Forecasting Weather*, p. 175. It is doubtful whether any hypothesis can rely upon air being still for dynamical purposes. There is always a slow drift even in the densest fog as mentioned elsewhere. G. I. Taylor has represented that the pattern of the eddy-motion of the atmosphere is independent of the mean velocity of the air-current to which the eddy-velocity is proportional Hence the motion in an anticyclone may be only a slow model of the same pattern in a stronger current but in the end causes the same thermal effect.

"fetch" over the North Sea. The cloud gets heavier in the Southern part of its course and not infrequently develops into a persistent drizzle of rain over Flanders and Northern France. Winds from the same quarter often give clear weather in the central and Southern parts of England with cloud on the Eastern coasts. We may perhaps attribute the difference to the enhancement of the coefficient of eddy-conductivity by the warmth of the water and to the increase of water vapour by evaporation from the sea.

A third example is to be found in the clouds of the Trade winds which take the form of strings of cumulus at a uniform level. According to Piazzi Smyth the level of clouds in the North East Trade wind at Teneriffe was 5000 feet, and the thickness of the Trade wind itself was about 10,000 feet, so that the clouds were in the middle height of the great current of air, not in its upper margin.

At St Helena in the heart of the South East Trade wind the mechanical effect of the Island itself, as an obstacle to the current, increases the amount of condensation; and to such an extent that near the top of the Island at 2000 ft. above sea level the air preserves an almost uniform humidity of 90 per cent. at 9 a.m. and the mean amount of cloud the year through at that hour is 85 per cent.

The particular type of cloud which is formed by the process which we have described depends upon the lapse rate of temperature in the region in which the condensation begins. If the lapse rate approaches that of the adiabatic for saturated air the initial condensation may give rise to cumulus heads, or even develop into a shower, whereas if the lapse of temperature is not near to that of the adiabatic for saturated air the cloud-layer must remain thin as the condensation will be dependent upon the forced vertical motion due to the eddies.

It seems possible that the scud which is often to be seen drifting in mid-air under a nimbus cloud after heavy rain is similarly due to the eddy-motion of the lowest layers operating upon the saturated air close to the ground. Eddy-motion is operative even with light winds, and when the lowest layers are completely saturated very small amounts of forced elevation due to turbulence would be sufficient to produce condensation where the eddies have some upward movement. It is noteworthy that the fragments of cloud here spoken of tend to arrange themselves at a definite level and it is possible that the remarkably level line along the slopes shown by morning clouds on a mountain side may be due to the regularity of the mixing of the surface layers in the eddy-motion of the slowly moving air. We have to remember that there can be no permanent cloud in perfectly still air. The lightest fog would settle if it were enclosed and kept still. Fog and cloud are always in a state of motion, sometimes only moving slowly but never still, and it would appear that the turbulence of the motion is necessary to keep the cloud in suspension.

In summarising this section let us remark that it has hitherto been usual to regard cloud as being associated with cyclonic weather and the upward convection of columns or limited masses of air. From the consideration of the inevitable effect of the churning of successive layers of air by the eddies which constitute the turbulence of currents of air moving over sea or land it becomes evident that if the process goes on unaltered for a sufficient length of time the

formation of cloud must occur, immediately at the surface if that is cold enough to give a mixture below the dew-point, in the upper layers if the surface is not colder than the air which flows over it; and in that way we may account for the formation of the many clouds which have been shown to have an inversion of the lapse rate of temperature within them and above them, and also of certain detached clouds formed like scud in exceptional positions.

Thus we may regard cloud in some frequent forms as being associated with currents of air of long fetch whether they belong to cyclones or anticyclones. Over the land the diurnal variation of temperature by affecting the eddy-motion introduces a corresponding diurnal variation of the conditions for the formation of cloud; over the sea the long travel of wind must end in cloud of some sort unless the diffusion upward of the eddy-motion is restrained by some lid of inverted lapse of temperature which confines the effect of the churning of the surface layers to the production of a shallow isentropic atmosphere in the lowest layers and the moisture of the surface layer is not sufficient to cause saturation within the stratum of turbulent air. Such is probably the daily experience of the air over the hot deserts of the tropical regions.

Föhn and Chinook Winds

There is another class of phenomena of weather for which the aid of the theory of eddy-motion must be invoked; these are the warm, dry, oppressive winds in the valleys on the lee-side of mountain-ranges. On the Northern side of the Alps such winds are well known as *föhn* winds and in the prairie country to the East of the Canadian Rocky Mountains as the *chinook*. It is a common practice to explain the hotness and simultaneous dryness of these winds by tracing the history of the air from low levels on the windward side through a period of rarefaction and reduction of temperature, as it is driven up the slope, culminating in the condensation of vapour and the formation of rain about the ridge, during which the great store of latent heat of evaporation is set free. By that time the air has acquired a greatly enhanced potential temperature, or an increased entropy as we may regard it, and the operations terminate in the realisation of this entropy in the raised temperature of the air when it reaches the lower levels again with its water taken away and a store of heat left in its stead.

This simple life-history is not quite satisfactory because there is no sufficient reason adduced for the air of the valleys on the windward side to climb up to the ridge, nor for it to get down again from the ridge to the valleys on the leeward side. Various reasons may be urged for some amount of climbing due to mechanical forces on the windward side, but there is nothing to be said in favour of the air at the ridge with its great store of entropy finding its way into the valleys as a warm dry wind except that the eddy-motion in the current which passes the ridge will gradually excavate the cold air from the valleys on the lee-side and ultimately the régime on the lee-side of the ridge will be a flow of isentropic air extending in thickness from above the ridge even to the deeper valleys. The cooling effect of the surface as the warm air reaches it will produce

a certain amount of stability in the lowest layers. But the process is not completed all at once. Some interesting details of its circumstances are given in von Ficker's papers[1].

Eddies and the General Turbulence of the Atmosphere

Before bringing our consideration of eddy-motion to a close we will call attention to two examples that will enable the reader to carry in his mind a general idea of the state of turbulence which exists in the atmosphere wherever an air-current passes along a boundary surface with a density different from its own, or, indeed, presumably wherever a discontinuity of velocity is associated with a discontinuity of density, for it is not the particular difference of density between water and air, or between the ground and the air above, that causes waves in the water and eddies in the air, but the existence of a discontinuity in the density, which may be called infinite when the boundary is solid, very large when the boundary is water, and very small, though still operative, when the boundary is a distinctly heavier gas. The process of the formation of eddies in such cases is most easily seen in the case of a liquid passing an

FIG. 7. A succession of eddies formed in a current of water passing a plane obstacle. (National Physical Laboratory.)

obstacle. This case is represented in an illustration taken from a report of the Advisory Committee for Aeronautics[2]. It will be seen that as the current passes the plate which forms the obstacle a succession of incomplete eddies is formed at regular intervals in the current. They travel along with the current and gradually disintegrate, filling that portion of the stream with irregular eddy-motion. A succession of obstacles would imply a corresponding series of disintegrating eddies having some recognisable form near the obstacle which causes them; in the further distance they have no defined form but merely fill the current with irregular turbulence. As the velocity of the stream is increased the eddies are formed at shorter intervals until a practically permanent eddy is formed at the obstacle itself.

[1] Heinz von Ficker, *Innsbrucker Föhn-studien*, Wien. *Denkschr. Ak. Wiss.*, vol LXXVIII, p. 83, 1905.

[2] *Report*, 1909–10. Reports and Memoranda, No. 31, fig. 7. Block lent by H.M. Stationery Office.

A permanent eddy of this kind is formed at the edges of cliffs or the ridges of houses or walls in all strong winds. When the wind blows upon the steep face of a cliff an observer standing at the edge may be effectually screened from the direct wind by the deflexion of the current upward in the formation of the eddy and he may feel only the return current which comes towards the edge of the cliff completing the circulation in the interior of the eddy. The writer recalls a remarkable example of a westerly gale at Dover in the spring of 1889 when the top of the Admiralty pier, apparently exposed to the full force of the wind, was the only place in Dover where it was possible to walk without discomfort on account of the violence of the wind. The protection was quite absent from the part of the pier where it joined the land and where the wind could travel up the slope of the beach without forming an eddy of the same size as that due to the nearly vertical wall.

When the air passes out to sea along a level surface at the top of a cliff a well-marked permanent eddy is formed on the face of the cliff. Photographs illustrating the course of a balloon in an eddy thus formed are included in the collection of photographs at the Meteorological Office.

The reader can make experiments for himself, simply with an empty match box or even his own hat, in the eddy formed by a strong wind blowing upon a nearly vertical cliff. A most remarkable example of a cliff-eddy can be found at the Rock of Gibraltar when a strong levanter blows on the steep Eastern face of the rock. Its effect upon the tube-anemometer which has been maintained at the signal station on the Rock is very remarkable. When the velocity of the wind reaches a certain limit it passes the opening of the anemometer in a direction nearly vertical and the effect is a reduction of pressure in the recording float. A limit is thus fixed to the velocity which the instrument can record and gusts of greater velocity appear on the record as entirely fictitious lulls, due to the withdrawal of the pen to the zero line by the "suction" of the air passing the anemometer. The sheet of air which forms the eddy in this case goes upward for some hundreds of feet. The phenomena were investigated by H. Harries of the Meteorological Office by means of small balloons and balls of cotton wool during a visit to Gibraltar. A description was contributed to the Royal Meteorological Society and to the discussion of a paper before the Aeronautical Society[1] in January, 1914.

For the eddy-motion of good ordinary meteorological exposure G. M. B. Dobson has made a careful study of the motion of the air in the lowest layer over Salisbury Plain at Upavon by following the motion of pilot-balloons with "no lift" set free near the ground and plotting the results[2]. Complete eddies are not at all conspicuous in the results but one aspect of the motion disclosed by some of the balloons with no lift must be mentioned here, though it breaks the continuity of our line of thought, because it shows that another problem must be faced in our study of the motion of surface layers. When

[1] Harries, *Q. J. Roy. Met. Soc.*, vol. XL, p. 13, 1914; Shaw, 'Wind Gusts and the Structure of Aerial Disturbances,' *Aero. Soc. Jour.*, 1914, p. 172.

[2] Advisory Committee for Aeronautics, Reports and Memoranda (New Series), No. 325.

the drifting balloon reached the edge of the slope leading down to the valley it was carried upwards along a continuous sloping line through as much as a thousand feet of height thus suggesting a sheet of air rising obliquely from the surface across the horizontal flow of the layers above, which would represent the wind on either side of the ascending sheet. This occurred in sunny weather when the surfaces both of the slope and the plain would supply heat to the air in contact with them. It would appear that the convection was localised in the sheet of air ascending obliquely from the lip of the slope and it is difficult to see how the continuity of the horizontal motion on either side of the sheet of rising air could be maintained. We may form an idea of the problem by supposing continuous lines of traffic, say four abreast, to be passing along Piccadilly straight on along Coventry Street and the line of vehicles on the off side to cross the traffic and pass up Shaftesbury Avenue at an angle of 45° to the continuous flow without any interruption of the continuity on either side. It could be arranged if at the proper moment corresponding vehicles in two consecutive rows changed places, and presumably something of that kind must occur in the air if a sheet of rising air, and not a narrow column is the proper mental picture of the conditions.

It may seem to the reader inappropriate that these remarks as to the nature of eddies in the atmosphere should come at the end of the section of this work which is devoted to the consideration of turbulence instead of at the beginning of the subject, but that course has been adopted with due deliberation. The turbulence to which the mathematical theory applies is no more a collection of eddies of definite size and shape than the molecular motion which accounts for the diffusion of heat and momentum according to the mathematical laws of conduction and viscosity is a collection of definite velocities belonging to the molecules. The air in motion over obstacles is full of turbulent motion belonging to eddies with a large range of size and in all directions just as the ordinary air-space is filled with molecules which have motions in all directions and with a large range of velocity. According to Taylor[1] the pattern of the eddy-motion is the same for all mean velocities of the wind, and the eddy-velocity increases proportionally with the mean velocity.

The reader may form a good idea of what is going on by watching what happens to the smoke which issues from a tall chimney in a strong wind. It is obviously in a state of turmoil but gradually spreads out laterally and vertically by the action of the eddy-motion in the air. If we imagine the process represented by the trail of smoke to be continued for a distance of some hundreds of miles we can form an effective idea of the result of the spreading upwards of eddy-motion due to turbulence, and we may also find some instruction in the superficial analogy between the trail of smoke from a factory-chimney and the trace of a tube-anemometer. If the reader will imagine a factory-chimney with its top in the trace of figure 2 of this chapter at the point indicated by "midt" he will find the trace on the left very suggestive of a smoke trail. We will leave him to think out for himself how far the superficial analogy has a

[1] Advisory Committee for Aeronautics, *Report*, 1916–17. Reports and Memoranda, No. 296.

real significance. At first sight it looks as though the chimney was an essential factor in the production of the eddies which appear to spread from it, but the eddies are already in the atmosphere and the smoke only makes their effect visible. They are not due to the motion past the chimney. Mr A. Mallock has usefully pointed out that the trail of smoke left by a steamer travelling through still air is not disturbed by eddies. There is ample relative motion of the funnel with respect to the surrounding air but there are no eddies in the air.

Consequently we must regard the properties of eddies in turbulent air in the same statistical manner as we regard the properties of the molecules of a gas. The analogy between the theory of eddy-motion of the atmosphere and the molecular theory of a gas is so close that the two may be regarded as subject to the same laws but with a large difference of scale. To fix our attention upon an individual eddy would be the reverse of helpful. Let us, therefore, regard the facts as illustrations rather than as the foundation of the mathematical theory which requires conditions of its own. Further investigation will disclose more effectively what these conditions are.

Notes. With reference to clouds in winds of long "fetch":

1. L. H. G. Dines of Valencia Observatory, Cahirciveen, has written that at Valencia with a North West wind which travels over successively warmer water, the weather is almost always of a violent squally type with vigorous convection to a height of 10,000 ft. or so and violent showers, the air between the showers being comparatively dry.

2. It may be remarked that the cooling of the air in the higher levels indicated by the formation of cloud, in consequence of the mixture of layers by eddy-motion, must have as its counterpart the warming of the air near the surface, and we may thus account for the relative warmth of the eastern side of Britain in a persistent WSW wind which was remarked upon in *Nature* by H. Harries, Jan. 9 and W. H. Dines, Jan. 16, 1919. The dynamical effect has been estimated quantitatively by Lieut. John Logie, R.A.F.

CHAPTER VI

The variations of wind with height in the upper air disclosed by observations of pilot-balloons[1]

FOR the first stage of our inquiry into the validity of the first law of atmospheric motion we have devoted our attention to the relation between the observed wind and the geostrophic wind at the surface where the disturbance of the relation between the wind and the distribution of pressure is certainly very considerable, and to the variation with height of the direction and velocity of the winds in the lowest layers. We have found that the observed phenomena in relation to these matters are the natural consequences of the eddy-motion of the surface-layer and that the wind gets more nearly in accord with the gradient as the effect of the eddy-motion becomes less marked. We have laid down no specification of the range of the lower layers. We have taken the first kilometre as being probably affected by the turbulence. We have given figures for variation of velocity up to 500 metres without any particular stress upon the selection of that level. The correlation coefficients between the deviations of temperature and pressure referred to on page 26 indicate 2 kilometres as the limit of disturbance for the middle of England in winter and 3 kilometres in summer.

From the nature of the method which has been followed the natural course would be to regard the position at which the geostrophic wind is attained, if we knew it, as marking the limit of the disturbance caused by the surface, and it has been the practice of the Meteorological Office to quote the value of the geostrophic wind determined from the isobars of the daily maps as probably representing with sufficient accuracy the actual wind at the level

[1] Observations of pilot-balloons are very numerous. The number of observations has increased very largely during the war. The results referred to in this chapter are derived from the discussions of various authors for which references are given in the text. Records of additional observations are to be found in various official publications: in the *Geophysical Journal* (M. O. Publication, No. 209 d, etc.), in the official publications of the Royal Prussian Aeronautical Observatory, Lindenberg, in a special daily publication of the Italian Meteorological Institute, in the publications of the Weather Bureau of the United States, and in the monthly publications of the International Commission for Scientific Aeronautics. A summary of the observations with a single theodolite at Aberdeen by A. E. M. Geddes, is given in a publication of the University Press of Aberdeen, 1915. The observations with kites which previously formed the chief basis of our exact knowledge of the winds in the upper air up to the level of about three kilometres were discussed by E. Gold in *Barometric Gradient and Wind Force* (M. O. Publication, No. 190, 1908), and in *Geophysical Memoirs*, No. 5 (M. O. Publication, No. 210 e, 1911). The reader may legitimately complain that the illustrations in this chapter are restricted to observations in the British Isles but the structure of the atmosphere in general is so complicated that in order to keep in mind some unity of ideas it seemed best to deal in the first instance with the problem as localised by a selection of the whole number of available observations.

of about 1500 or 2000 feet or 500 metres. But at this stage we are not in a position to go further than that in defining the limit of interference of the surface with the free course of the air, because the balance which we postulate is between the wind and the distribution of pressure at the same level. In order to make the comparison with the wind at 1500 or 2000 ft. we ought to obtain the distribution of pressure at those levels. Before approaching that part of the subject it will be best to examine the variations of wind with height in the levels which we may suppose provisionally to be for practical purposes free from the disturbing influence of the turbulence due to the surface, in order to obtain some guidance as to the changes to be expected from other causes than eddy-motion.

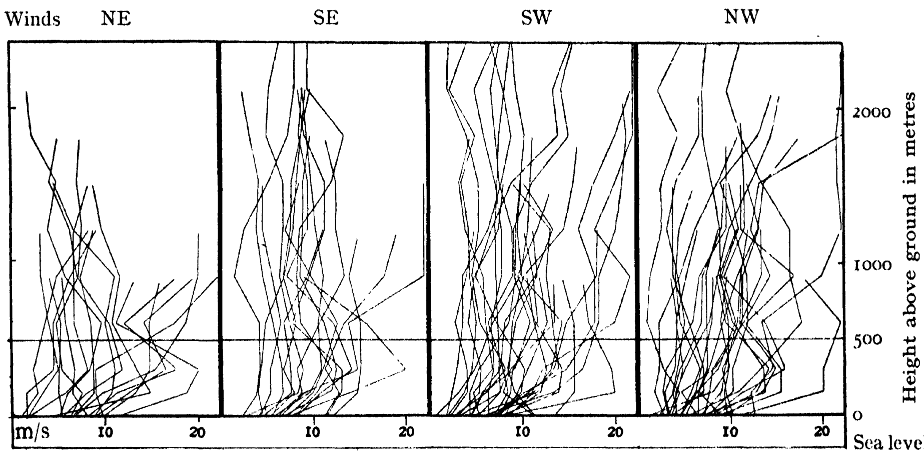


FIG. 1. Change of wind-velocity with height at Upavon, between the level of 2500 metres and the surface, for winds in different quadrants.

We shall find the structure of the atmosphere from this point of view extremely complicated and we must seek the probable cause of these complications. As giving a general representation of the different cases that arise we will take the observations with pilot-balloons made at Upavon by G. M. B. Dobson[1] and discussed by him in a paper before the Royal Meteorological Society in 1914. It gives an excellent summary of results for the layer of air from the surface to the level of nearly 2500 metres (8000 ft.) at a well-exposed station. This height includes the surface-layer and gives also an insight into the variations through a suitable range for the next stage. The soundings are 97 in number, the great majority being followed with one theodolite only. The site of the observatory is 183 metres (600 ft.) above sea level and the situation is very favourable because it gives a large expanse of nearly level plain.

[1] G. M. B. Dobson, 'Pilot Balloon Ascents at the Central Flying School, Upavon, during the year 1913,' *Q. J. Roy. Met. Soc.*, vol. XL, p. 123, 1914.

From Dobson's paper we take four composite diagrams representing the individual ascents for winds in the four quadrants, 18 in the North East, 18 in the South East, 33 in the South West, and 29 in the North West respectively. We have added to the diagrams as published in the original paper a full line at 500 metres and also the line of sea level. The line at 500 metres divides the height of the ascents into two parts. Within the lower part is the rapid increase of the wind in the layer at the surface which is characteristic of all the winds and shows generally the effect of the surface-turbulence. Yet there are cases in all the quadrants, and particularly in the North West quadrant, in which the first step from the surface shows a diminution of velocity. We shall refer to this peculiarity later, on page 65.

Looking at the appearance of the curves in the composite diagrams the direct effect of the surface seems to have come to an end before the level of 500 metres was reached. We may also notice as a general feature the remarkable irregularities that are disclosed in all the quadrants. We may fairly say that the winds in the North East quadrant fall off with height beyond 1000 m., that those in the South East quadrant remain steady[1], those in the South West and North West quadrants show all variations from approximate uniformity to large increases of velocity in the upper levels, but above and against all these general statements must be written that exceptions do occur and the variations on different occasions follow no absolute rule. That is the situation which we have to face in dealing with the observations of winds in the second stage between 500 m. and 2500 m.

Numerically the results are summarised as follows: as regards velocity, on the average the geostrophic wind corresponding with the surface gradient was just reached at 915 metres *with N.E. winds* and then the velocity *began to diminish. With S.E. winds* the velocity reached the geostrophic wind below 300 metres and *kept quite close to it* from that level upwards, *with S.W. winds* the calculated value was reached near the 500 metre-level and thereafter the velocity *increased to* 117 *per cent.* of the geostrophic wind at about 2500 metres and *with N.W. winds* the geostrophic wind was reached below 300 metres and thereafter *the velocity increased* at 8000 ft. (2500 metres) *to* 145 *per cent.* of the calculated value. On the other hand, as regards direction, N.E. winds, starting with an average deviation of 27° at 50 metres, only got within 6° of the line of the surface isobar even at the highest point tabulated; S.E. winds with an initial deviation of 24° veered to the isobar at 600 metres and passed beyond it. S.W. winds in like manner starting from a deviation of 19° passed the line of the isobar at about 800 metres and carried the veer 16° further, and N.W. winds starting with a deviation of 11° kept close to the line of the surface isobar between 600 metres and 1200 metres and then veered 8° from it.

Grouped according to the velocity at 605 metres, light winds (below 4·5 m/s) showed little increase of velocity with height and do not seem to have

[1] This characteristic of winds in the South East quadrant is borne out by Cave's observations at Ditcham Park which are referred to later. Cave's class of "solid current" included a large number of examples in the South East quadrant.

quite reached the geostrophic wind below 1000 m. Moderate winds (between 4·5 m/s and 13 m/s) attained the geostrophic velocity at 300 metres and strong winds (above 13 m/s) at 500 metres.

For light winds the surface velocity is 87 per cent. of the wind at 650 m., for moderate winds 64 per cent. and for strong winds 56 per cent. Light winds also show considerably less change of direction with height than do moderate or strong winds. "It is remarkable that the moderate winds should go on veering considerably after the direction of the isobar has been reached."

These results as regards the relation of the surface-winds to the upper winds are in accordance with the conclusions which have already been reached in chap. IV, and details of the characteristics of the variation with height of the velocity and direction of strong winds have been used by Taylor, as we have seen, to verify his theory of the effect of turbulence. We have selected this particular group of observations partly on account of the favourable nature of the site and partly because they are based mainly upon the method of the single theodolite which is now in daily use at many stations. The results include any defects of the method but they present the problems which have to be faced in considering the observations from the stations where that method is employed.

As giving a summary of results obtained from seventy-three soundings made with special care, mainly by the use of two self-recording theodolites, we take from the third report of the Advisory Committee for Aeronautics[1] J. S. Dines's discussion of the results obtained from pilot-balloon-ascents at Pyrton Hill, 1910–11. The site is in very open country 150 metres above sea level on the western slope of the Chiltern Hills near Watlington in Oxfordshire. But it has, on that account, some disadvantages for work with pilot-balloons. "The surrounding hills subtend an angle of 10° above the horizon from N. round through E. to S. From S. the altitude falls off to zero at W., and from that point to N. the horizon is clear." The peculiarities of the site limited the range of observation with strong winds from the Western quadrants and protected the surface against strong winds from the Eastern side The observations are therefore confined mainly to days with a surface-wind below seven metres per second. Nor are all wind-directions represented. There are no observations from South East or South West. Their absence is accounted for partly by the site and partly by low clouds which are a common accompaniment of winds from those quarters.

The results for groups arranged according to the wind-directions and also for the means of all the observations at successive levels at stated hours are represented in the diagrams of fig. 2. They show the characteristic points of the results of soundings with pilot-balloons, the rapid increase of velocity near the surface with winds from any quarter, which ceases below the level of 500 metres for all the examples plotted except that of the winds grouped as WNW. There is a notable falling off at about 1000 metres of the velocity of

[1] *Report for* 1911–12. *Third Report on Wind Structure.* Reports and Memoranda, No. 47, 1912.

FIG. 2. STRUCTURE OF THE ATMOSPHERE from 100 metres to 2000 metres at Pyrton Hill. Mean Values of Speed and Direction of horizontal velocity of the wind and the vertical velocity of the balloon for soundings grouped according to the direction at the surface (J. S. Dines, 1910–11).

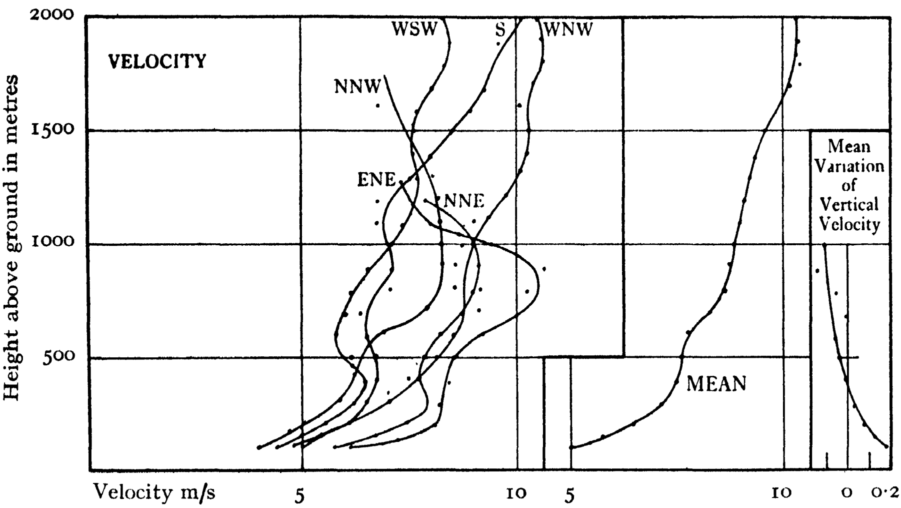


FIG. 2 *a*. Variation of the velocity of wind with height.

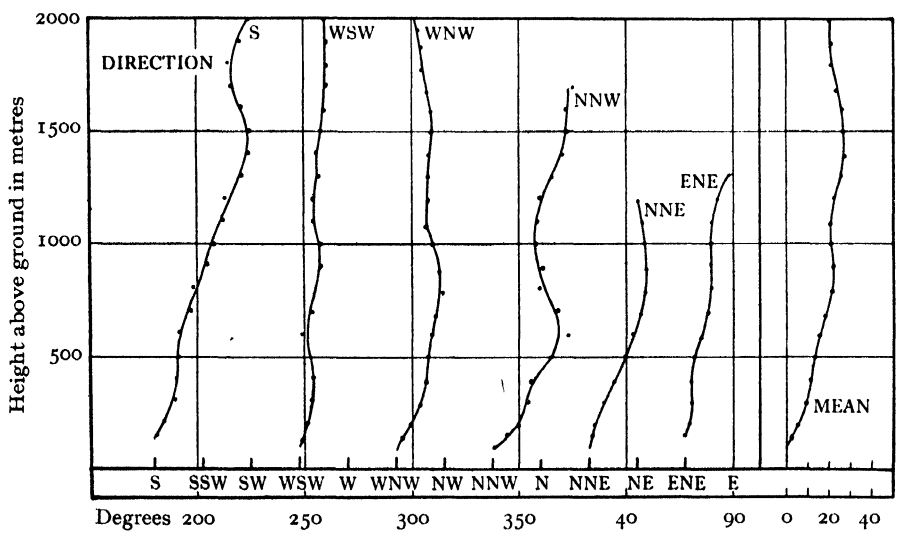


FIG. 2 *b*. Variation of the direction of wind with height.

the winds grouped as NNE and ENE, especially the latter, after a considerable further increase from the limit of the first increase of the surface-wind. The winds grouped as S and WSW show an irregular but generally continuous increase up to the level of 2000 metres which is most pronounced in the WSW group.

These characteristics are generally in accord with the results already quoted for Upavon on p. 57, but it must be understood that even when the observations are with two theodolites the mean values include examples which deviate a good deal in various ways from the mean. We cannot yet make any satisfactory generalisation as to the variation of wind with height above the level of 500 metres that can be regarded as applicable in all individual cases.

Before passing on to other examples of the structure of the upper air disclosed by pilot-balloons we may interpolate a note as to the relation of the winds at 50 metres (the lowest level of observation in the series now under consideration) with the geostrophic wind as determined from the maps of the Daily Weather Report for the observations at Pyrton Hill. We will also give the comparison of the results with those of the calculated ratio of W/G by G. I. Taylor's theory of the relation between the ratio W/G and the angular deviation α of the wind from the run of the isobar. The figures, which should be compared with those given with fig. 1 of chap. II, are as follows:

TABLE I. COMPARISON OF OBSERVED AND CALCULATED VALUES FOR THE RATIO OF THE WIND AT 50 METRES TO THE GEOSTROPHIC WIND.

		S	WSW	WNW	NNW	NNE	ENE	Mean
W (100 m.)	m/s	5·0	4·25	4·75	4·25	5·75	6·0	5·0
G (from D. W. R.)	m/s	9·5	9·5	7·75	7·75	7·5	12·5	9·0
W/G ("observed")		·55	·45	·61	·55	·77	·48	·56
α from D. W. R.		$17\frac{1}{2}°$	5°	5°	11°	30°	$22\frac{1}{2}°$	13°
$\cos\alpha - \sin\alpha$ W/G ("calculated")		·65	·91	·91	·79	·37	·54	·75

It will be seen that the agreement is fairly good for the S and ENE groups. For the others, the "calculated" value is much in excess of the "observed" value. There are various possible explanations of these differences which must be explored before a final opinion can be arrived at, and they are mentioned here in order to lay stress upon the fact that the comparison of the surface-wind with the geostrophic wind requires very close and careful observation if uncertainties of a difficult character are to be avoided. As we have already seen the diurnal variation of the wind is a matter of importance, so that it is necessary for the observed wind and the map from which the geostrophic wind is obtained to be properly synchronous and a suitable hour selected; and in the case of Pyrton Hill the peculiarities of the site may have had some influence. The South winds blow at about forty-five degrees to the line of the hills, those from WSW are climbing the hills, those from the

ENE have come over them. The effects of such circumstances as these upon the direction of the wind near the surface are as yet unexplored.

The Variations of Vertical Velocity of Pilot-Balloons

The great advantage of the method of two theodolites, especially when they are provided with means for recording continuously the altitude and azimuth of the balloon which is being followed, is that the observations enable the observer to determine the actual height of the balloon and thus obtain a measure of the variation of the vertical velocity. If the actual rate of ascent of the balloon in still air were exactly known, in spite of the solarisation of the balloon and other possible causes of change, the differences from the observed velocities would give the vertical component of the motion of the air at successive levels, but for dealing with small differences it is not safe to assume that the velocity of ascent in still air is sufficiently well known in ordinary cases.

In the discussion of the observations at Pyrton Hill J. S. Dines obtained the variation in the rate of ascent and the mean values of these variations are shown in a small inset in the diagram of fig. 2*a*. It appears from the curve that the balloons lost on the average about 0·3 m/s of ascensional velocity within the first kilometre and the greater part of the loss took place within the first half kilometre. In explanation of this general result, which agrees with what had been previously observed by Cave at Ditcham Park and by Hergesell at Strasbourg[1], J. S. Dines has pointed out in a note upon his discussion[2] that balloons set free at the surface will as a rule be carried by the current in the surface-layer away from the region where air is descending and towards the region where air is ascending, because ascending air must be supplied by currents moving over the surface from places where it is descending. The fact that evidence for this conclusion can be detected in the mean values of a large number of soundings with pilot-balloons is very satisfactory evidence of the general precision of the measurements. Dines pursued the question of the vertical component of air-currents within the lowest two kilometres still further and found a range of vertical velocity in the lowest kilometre amounting to 3·2 m/s on June 27, 1911, in a sky with a few small detached cumulus clouds, and a range of 3·0 m/s within the second kilometre on the same day. On July 5, of the same year, also in a sky with some cumulus, he found a range of 3·6 m/s in the second kilometre and summarising a table of results he adds "It appears from the records obtained that vertical currents (of short duration) of 2 m/s must be not uncommon on days with detached cumulus about, while on days of clear sky the current would not as a rule exceed ·5 m/s."

After satisfying himself about the application of a formula for the ascent

[1] *Commission Internationale pour l'Aérostation Scientifique*, Report of Meeting at Monaco, 1909, p. 102.

[2] *Third Report on Wind Structure*, Advisory Committee for Aeronautics, *Report*, 1911–12, p. 230.

of a pilot-balloon of given weight and lift[1], in a subsequent report[2] to the Advisory Committee for Aeronautics, not yet published, J. S. Dines has discussed the information about the vertical component of the motion of the air derived from 66 soundings with pilot-balloons at Pyrton Hill which were watched with a pair of recording theodolites. He has given the vertical component at different levels in each of the soundings and from his results (which include only 89 observations in the various levels, out of a total of nearly 1000, when the vertical component is set down as zero) a table of frequencies of vertical components within certain specified limits has been compiled, Table II.

TABLE II. FREQUENCIES OF VERTICAL COMPONENTS OF WIND-VELOCITY IN METRES PER SECOND AT DIFFERENT LEVELS FROM 66 SOUNDINGS WITH PILOT-BALLOONS AT PYRTON HILL.

	Downward			Upward											
Level	1·0 to 1·4	0·5 to 0·9	0 to 0·4	0 to 0·4	0·5 to 0·9	1·0 to 1·4	1·5 to 1·9	2·0 to 2·4	2·5 to 2·9	3·0 to 3·4	3·5 to 3·9	4·0 to 4·4	4·5 to 4·9	5·0 to 5·4	Number of observations
m.															
100	—	2	13	27	18	1	2	2	1	—	—	—	—	—	66
150	—	4	17	23	12	5	—	3	2	—	—	—	—	—	66
200	2	5	19	22	7	3	3	2	1	2	—	—	—	—	66
250	2	8	21	16	8	3	2	4	—	1	1	—	—	—	66
300	3	8	23	16	6	2	3	2	1	1	1	—	—	—	66
400	4	11	26	9	6	4	3	—	—	2	—	1	—	—	66
500	2	10	27	15	3	4	2	—	—	2	—	—	1	—	66
600	1	8	26	16	5	3	2	1	1	—	—	—	—	1	64
700	1	4	28	15	5	2	2	2	—	—	—	—	1	—	60
800	1	3	25	13	8	2	2	1	—	—	1	—	—	—	56
900	1	2	19	19	6	—	3	—	—	—	1	—	—	—	51
1000	1	4	15	15	4	3	1	1	—	1	—	—	—	—	45
1100	—	6	9	11	8	2	—	—	1	—	—	—	—	—	37
1200	1	3	12	9	8	3	—	—	—	—	—	—	—	—	36
1300	1	5	10	10	7	—	2	—	—	—	—	—	—	—	35
1400	1	4	9	8	6	—	1	1	—	—	—	—	—	—	30
1500	—	4	8	9	5	—	—	—	—	—	—	—	—	—	26
1600	—	2	11	9	2	—	—	—	—	—	—	—	—	—	24
1700	—	1	11	6	1	—	1	—	—	—	—	—	—	—	20
1800	—	2	10	7	—	—	—	—	—	—	—	—	—	—	19
1900	—	3	7	4	—	—	1	—	—	—	—	—	—	—	15
2000	—	4	5	5	—	—	1	—	—	—	—	—	—	—	15
	21	103	351	284	125	37	31	19	7	9	4	1	2	1	995

The table shows that an upward vertical component of 5·1 m/s was registered on one occasion at the level of 600 m. and components between 4 m/s and 5 m/s three times between 500 m. and 1100 m. The maximum components downward are not so large but reach a limit greater than 1 m/s and less than 1·5 m/s on several occasions.

[1] *Q. J. Roy. Met. Soc.*, vol. XXXIX, p. 101, 1913.

[2] *Fifth Report on Wind Structure.* Reports and Memoranda, No. 95.

The gradual change with height in the frequency of upward motion and downward motion is noteworthy. At the level of 100 metres there are fifty-one rising currents as against fifteen downward and the distribution gradually changes until at 500 metres, the last level for which the full number of sixty-six soundings are available, there is a preponderance for descending motion of thirty-nine as against twenty-seven. But a preponderance of upward motion shows itself again at 900 metres and is continued to 1500 metres with a maximum at 1100 metres.

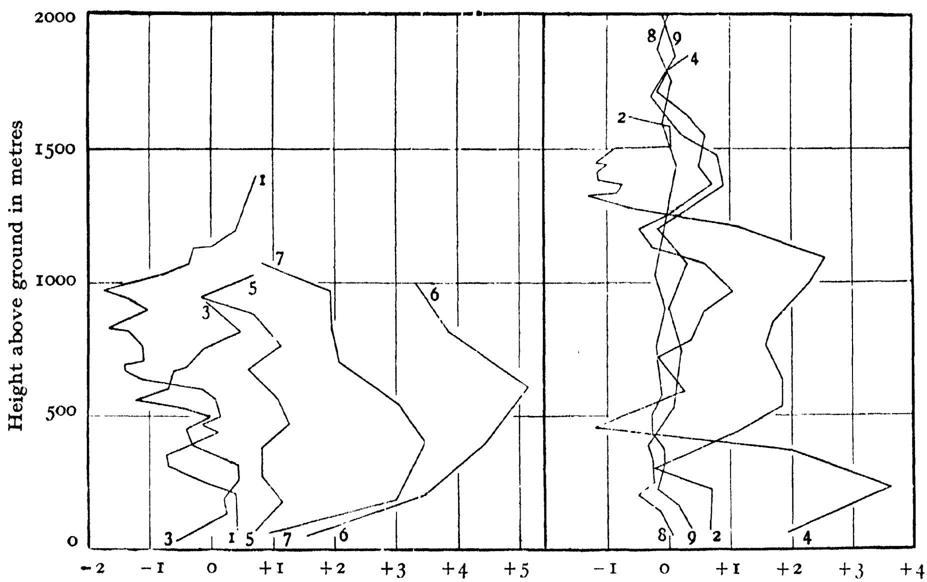


FIG. 3. Vertical components of air-currents at Pyrton Hill. The velocities are marked, below the diagrams, in metres per second, upward components +, downward components −.

For comparison with the corresponding weather-maps numbers have been inserted on the diagrams to show the dates and times of the observations as follows:

1.	1911	May 4	11 h. 35 m.		5.	1913	May 10	12 h. 10 m.
2.	1912	April 30	19 12		6.	1913	May 22	10 55
3.	1912	May 9	11 51		7.	1913	May 26	11 10
4.	1912	June 22	11 12		8.	1913	Oct. 13	14 50

9. 1913 Oct. 23 15 h. 40 m.

The general results may be somewhat biassed in favour of the display of vertical motion because the occasions were chosen with a view to exploring cases of vertical motion and do not properly represent random sampling.

Diagrams illustrating the rapid variation of vertical velocity with height are given in fig. 3. Such currents are regarded by J. S. Dines as only transient because in those cases when a sounding was repeated after a short interval of half an hour or less they were not recorded.

The importance of these determinations of vertical components in the stratum up to the two-kilometre level lies in their bearing upon the observations with a single theodolite in the computation of which the vertical component of the air-velocity is ignored; but if there is a vertical component it will affect the determination of the horizontal velocity by the single theodolite and a correction will be necessary. An ascending current will cause the computed velocity to be too small and might even cause an apparent reversal of the wind, and conversely a descending current will exaggerate the speed of the balloon away from the observer and in certain circumstances may give very inappropriate results.

The following computation of the correction to the computed velocity u_0 away from the observer for a vertical component w is due to Dr H. Jeffreys.

E is the angular altitude of the balloon, z its height, x its horizontal distance, A the azimuth of the balloon. The components of the velocity of the balloon are $\dot{z}$, upwards, $\dot{x}$ horizontal in the direction of the projection of the line of sight, and $x\dot{A}$, transverse to the line.

Then $\dot{z} = Z + w$ where Z is the vertical velocity of the balloon in still air.

Then $$x = z \cot E,$$

$$u = \dot{x} = \dot{z} \cot E - z \operatorname{cosec}^2 E\, dE/dt$$

$$= (Z + w) \cot E - z \operatorname{cosec}^2 E\, dE/dt.$$

In computations from observations with a single theodolite it is usual to assume that w is zero. Thus the computed velocity is too small by $w \cot E$ along the projection of the line of sight. The height z also is in error by a calculable amount depending on the values of w during the earlier parts of the ascent. Hence, to correct for the vertical component of velocity, w, the term $w \cot E$ should be added to the computed velocity along the line of sight.

The amount of the correction depends upon the altitude of the balloon as observed in the theodolite. It requires to be borne in mind in considering the results derived from observations with a single theodolite. Thus, for example, the initial loss of velocity on leaving the ground which we have noticed in some of the individual curves included in Dobson's diagrams might possibly be attributable to ascending currents near the ground which are quite likely to be present in the daytime.

A similar explanation may be suggested for exceptional velocities, either high or low, which are sometimes obvious when the observations of pilot-balloons with a single theodolite come to be plotted on a map. Such an exceptional velocity is often shown in the observations for 1000 ft. and 2000 ft. from Barrow-in-Furness as compared with those at surrounding stations. It is possible that the peculiar position of the station with reference to the hills of the Lake District may be the cause of descending or ascending

currents, at the level of 1000 or 2000 ft. which will be different according to the direction of the wind.

The two examples which have been selected for illustrating the results obtained by pilot-balloons are both dependent upon inland sites and differ the one from the other merely in the fact that the station at Upavon is on a

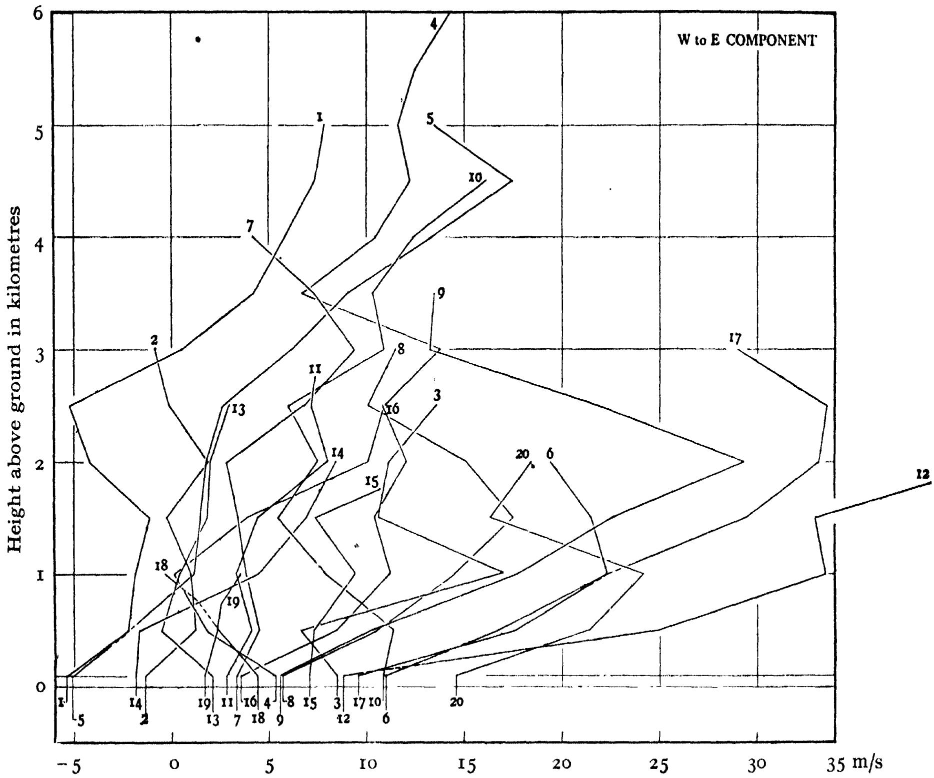


FIG. 4 *a*. Structure of the atmosphere below 6000 metres on the East Coast (Geddes). W to E component of velocity.

Numbers are inserted in the diagrams 4 *a*, 4 *b*, 4 *c*, to show the dates of the observations as follows:

1.	1912	May 22	6.	1913	Oct. 17	11.	1912	June 21	16.	1913	Nov. 1
2.	1912	June 26	7.	1913	Nov. 5	12.	1913	Dec. 13	17.	1913	Nov. 26
3.	1912	Nov. 22	8.	1913	Nov. 14	13.	1913	Feb. 28	18.	1912	June 7
4.	1912	Nov. 29	9.	1913	Nov. 19	14.	1913	May 16	19.	1912	June 14
5.	1913	June 13	10.	1913	Dec. 5	15.	1913	June 11	20.	1912	Nov. 20

The time of ascent was, in every case, between 11 h and 12 h.

level plateau whereas that at Pyrton Hill is on the North West slope of a range of hills which runs from South West to North East. As a third example we may take the observations by A. E. M. Geddes[1] at Aberdeen because they are derived from a station on the coast. A station in such a position may be

[1] *Q. J. Roy. Met. Soc.*, vol. XLI, p. 123, 1915.

expected to have special characteristics as regards the structure of the first two kilometres of its atmosphere because on the one side is the land with all the disturbances due to the irregularities of relief and variations of temperature at the surface, and on the other side is the sea, the surface of which is free from those irregularities and variations. We may, therefore, expect the results

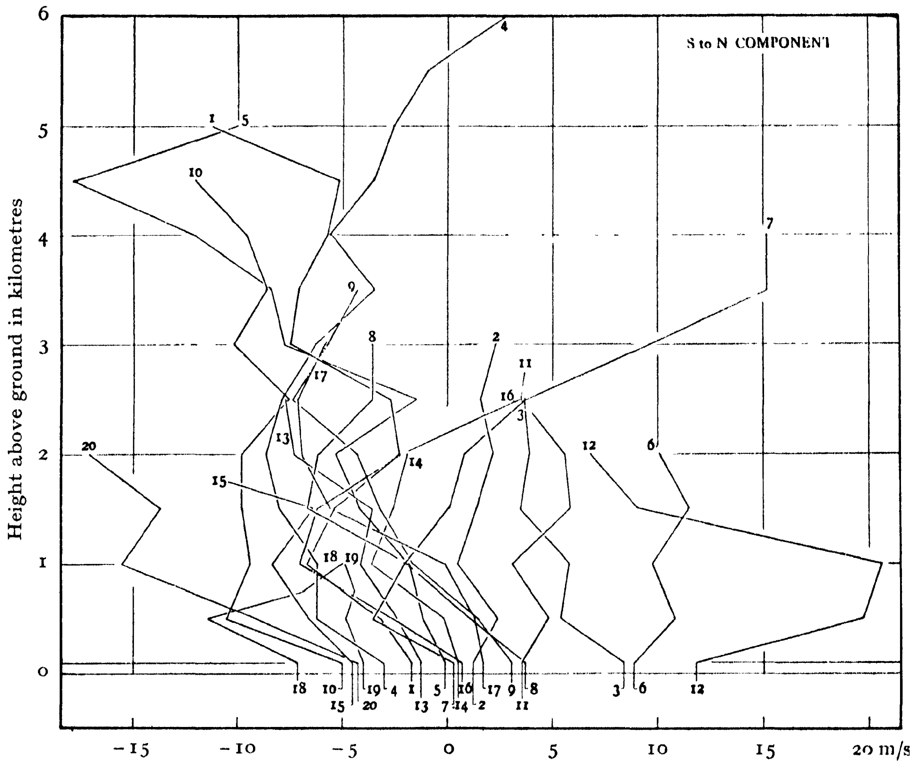


FIG. 4 *b*. Structure of the atmosphere below 6000 metres on the East Coast (Geddes). S to N component of velocity.

of soundings with pilot-balloons to display complications even more involved than those which are displayed at Upavon or Pyrton Hill. The expectation is certainly realised in the diagrams which represent the components of the motion of the air at different levels up to 2500 metres as taken from the tables given in Geddes's paper. Two theodolites were used for the observations and therefore the influence of the vertical component upon the velocity of ascent can be represented by the variation of the vertical velocity of the balloon which is given in the tables and is also represented in the diagrams.

Geddes groups his observations according to the fate which ultimately overtook the balloon and the examples represented in the diagram are for

those lost in haze or distance and those lost in strato-cumulus cloud with three examples when stratus clouds terminated the observations.

The reader will note that in these diagrams components have been plotted instead of the resultant velocity with its direction. The use of components is desirable partly because it may simplify the examination but mainly because whenever computations have to be made with the view of combining measures of wind it is nearly always necessary as a first step to resolve them into their components. In all cases the student who wishes to comprehend the structure

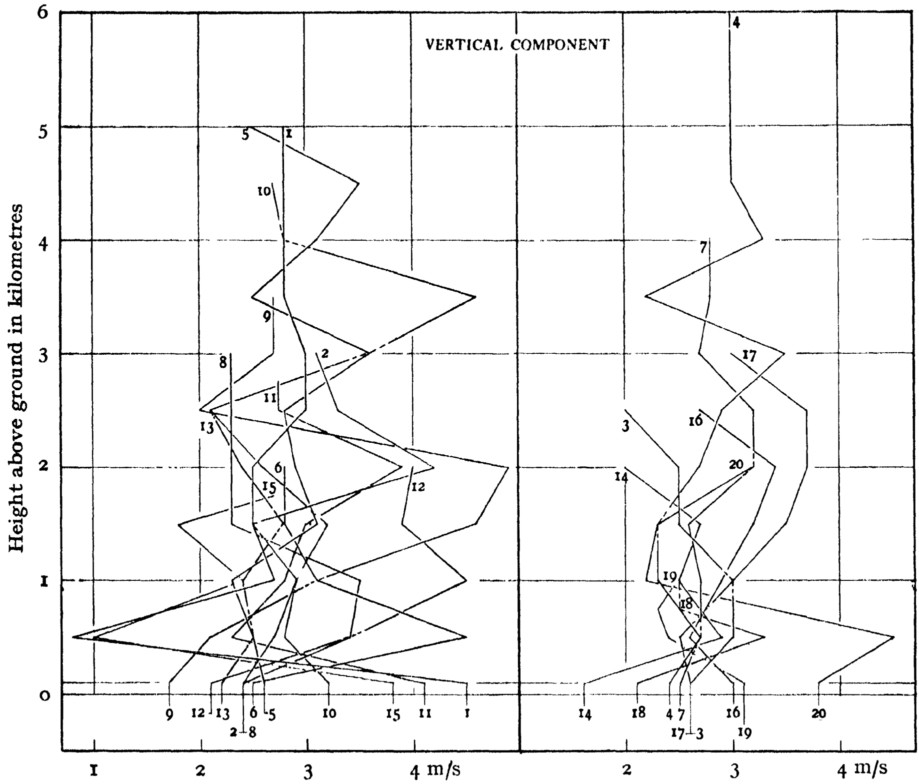


FIG. 4 *c*. Structure of the atmosphere below 6000 metres on the East Coast (Geddes). Vertical velocity of the Balloon, showing changes with height of the vertical component of the velocity of the air.

of the atmosphere has to read simultaneously two diagrams and for any comprehensive study the use of components will ultimately be found the more satisfactory alternative. The process is also desirable in view of the difference of geographical significance between motion along parallels and motion along meridians on the earth's surface. At Aberdeen the difference must be considerable because the North Sea lies to the East and mountains to the West. Perhaps a more definite result in that particular case might have been obtained by resolving along a North East and South East line instead of due North and South.

The illustrations which have been adduced in this chapter are sufficient to set before us the complexity of the problem presented by the facts regarding the structure of the atmosphere between the levels of 500 and 2500 metres[1]. It is probably the layer of greatest complexity. It includes not only the varieties of structure incidental to the kinematics of the free atmosphere in its simplest form depending on the relations of its motion to the distribution of pressure and temperature, but also some disturbance due to the effect of eddy-motion wherever circumstances are such that a high value of the coefficient κ of eddy-conductivity has been operative for a long period. There is also to be found the complication due to vertical motion of which the upward tendency is most marked in these strata but with which the compensating downward motion must be recognised and, with those, the consequent local disturbance of the horizontal motion.

We now pass on to lay before the reader an account of another section of the atmosphere, that between 2500 metres and 7500 metres as representing the layers which we may be encouraged to regard as somewhat less complicated in their structure because they represent the region where according to W. H. Dines's results the deviations of pressure for any level from their normal show a close approach to proportionality to the simultaneous deviations of temperature; and in the ordinary conditions, when soundings with pilot-balloons are practicable, there is at least comparative freedom from vertical motion. These two specifications may be regarded as interdependent. If the motion of the air is confined to horizontal layers the changes in the temperature of the air will necessarily be governed by the changes in pressure, provided we may leave out of account such slow changes as are attributable to the loss or gain of heat by radiation and express themselves perhaps in the seasonal variations of temperature at the different levels.

In order to represent the structure which has been observed in the layer which extends from 2500 metres to 7500 metres we will make use of the results obtained by C. J. P. Cave principally at Ditcham Park which lies at a height of 167 metres, close to the ridge of the South Downs in Hampshire. In his book[2] "smoothed" results are given for 200 soundings in the years 1907 to 1910, and represent the first investigation of the upper air by means of pilot-balloons in this country. For some of the soundings two theodolites were employed, for others only one. We have drawn no distinction between them in the selection made for our purpose because for exploring the layers under consideration the two methods are about equally effective[3]. From Cave's tables we have taken all the ascents which gave smoothed values for each half-kilometre from 2500

[1] The complication is even more irregular in other countries where the thermal changes and the geographical relief are more pronounced. A report on the winds of Macedonia as ascertained by soundings with pilot-balloons presented to the Advisory Committee for Aeronautics (*Report*, 1916–17, Reports and Memoranda, No. 296) illustrates this statement.

[2] C. J. P. Cave, M.A., *The Structure of the Atmosphere in Clear Weather*, Camb. Univ. Press, 1912.

[3] See some remarks upon the comparative errors of the two methods by G. M. B. Dobson *loc. cit* p. 130.

metres to 7500 metres and have included also three which only missed the last step. There were 23 soundings in all and of these 22 have been plotted in a diagram to represent the West to East components of the wind-velocity and 18 for the South to North components. Points to the left of the zero line represent Easterly or Northerly winds and those to the right Westerly or Southerly respectively. The soundings have been omitted from the diagrams only when

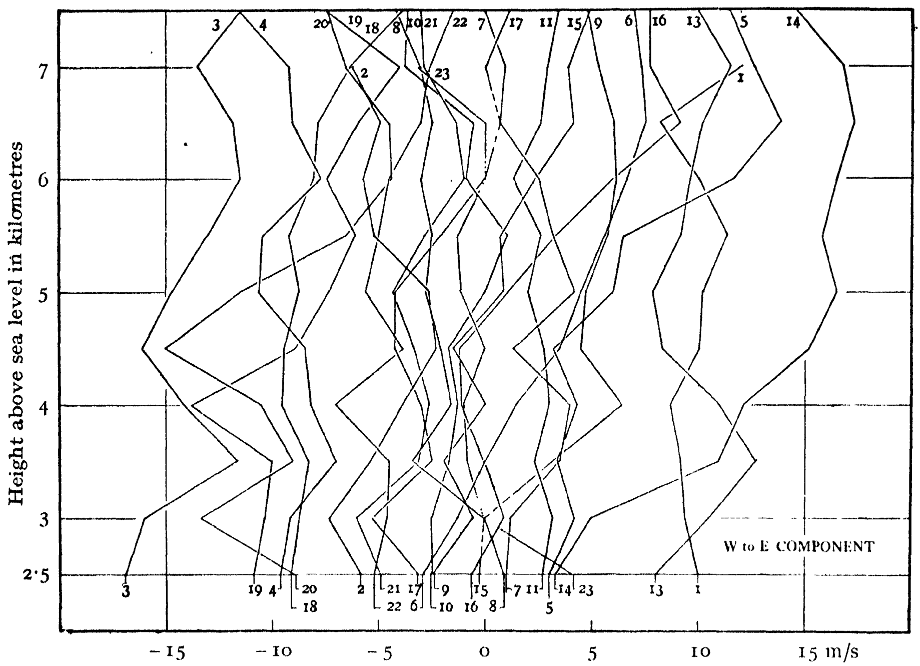


FIG. 5 *a*. Structure of the atmosphere from 2500 metres to 7500 metres at Ditcham Park (Cave). W to E component of velocity.

The numbers assigned to the lines in the diagrams show the dates of the observations as follows. The letters prefixed show the class as indicated on p. 72.

a 1.	1908	May	18	*e* 7.	1907	Sept.	23	*e* 13.	1908	July	30	*c* 19.	1909	May	7
a 2.	1908	May	30	*e* 8.	1908	June	22	*e* 14.	1908	July	31	*c* 20.	1909	May	7
a 3.	1909	May	5	*e* 9.	1909	Feb.	18	*b* 15.	1908	Sept.	30	*a* 21.	1909	Aug.	5
a 4.	1909	May	6	*e* 10.	1908	June	3	*b* 16.	1908	Oct.	1	*c* 22.	1910	Mar.	3
d 5.	1908	July	27	*e* 11.	1908	July	28	*b* 17.	1908	Oct.	2	*b* 23.	1907	May	24
d 6.	1908	Nov.	6	*e* 12.	1908	July	29	*a* 18.	1909	May	6				

Particulars of the soundings are given in Cave's *Structure of the Atmosphere in Clear Weather*.

practically they repeated types already shown or when they belonged to a part of the diagram that was already sufficiently filled.

In these diagrams we may begin to see some suggestion of order. In that representing the West to East components the range of velocity at each level is approximately the same but on the whole the Westerly components are stronger at the higher levels. Thus at 2500 metres the range is from $-16{\cdot}5$

which means an East component of 16·5 m/s to + 10 which means a West component of 10 m/s, whereas at 7500 metres the range is from – 11·5 to 15 so that the East component has lost 5 m/s and the West component has gained 5 m/s at the higher level. The particular figures are perhaps accidental, but the general trend towards Westerly winds at higher levels is a real phenomenon.

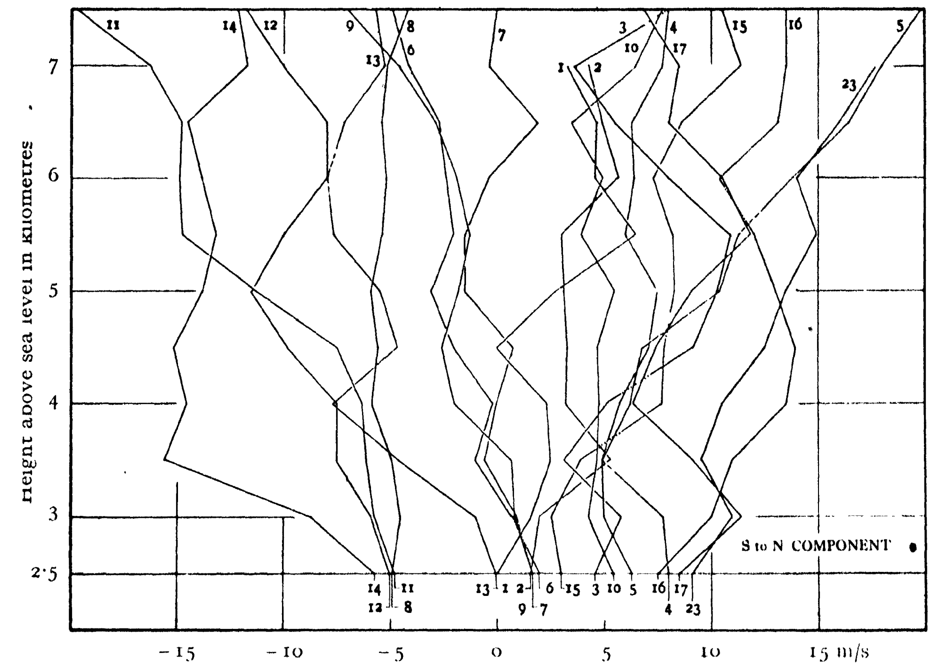


FIG. 5 *b*. Structure of the atmosphere from 2500 metres to 7500 metres at Ditcham Park (Cave). S to N component of velocity.

In the diagram representing the South to North components, on the other hand, we notice that the range of velocity is much wider at 7500 metres than it is at 2500 metres. It extends from – 20 which means a North component of 20 m/s to + 20 which means a South component of the same magnitude. The range at 2500 metres is indeed very small being from only – 6 to + 9. It must not be supposed that the figures quoted from the diagrams mark the practical limits of velocity of winds, or their components from North or South, East or West, at the level of 2500 metres. A more reasonable inference is that the occasions when higher wind-velocities might have been experienced at that level were not suitable for a sounding up to the higher level of 7500 metres, on account either of the formation of cloud or the difficulty of keeping a small balloon within observation in a strong wind.

Captain Cave has grouped his soundings into five classes or types of structure within the troposphere, namely (*a*) those which show "solid current" or little change from the direction and velocity at the surface over a large

range of height, (*b*) those that show a considerable increase of velocity without much change of direction, (*c*) those which show a decrease of velocity with height, (*d*) those which show a reversal or great change of direction in the upper layers, and (*e*) those which show an upper wind (either in the North West quadrant or South West quadrant) crossing the lower wind and therefore coming apparently from above the central region of the cyclonic depression of which the South Westerly wind occupied the Southern sector; in reality they are doubtless circulating round a low pressure centre to the North East or North West. The classes to which were assigned the soundings used in forming the diagrams are marked by letters in the list of dates. Of the whole number of the 23 ascents, six are in class (*a*) "solid currents" showing little change with height, four in class (*b*) steadily increasing currents without much change of direction, three are the diminishing currents of class (*c*), two are reversals, of class (*d*), and eight are in class (*e*), increasing cross upper currents generally from North or North West.

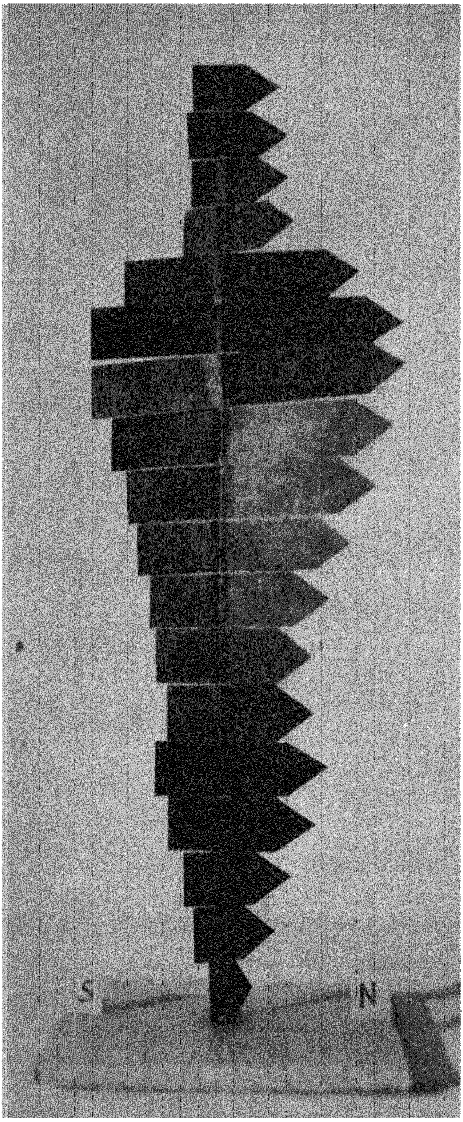


FIG. 6. Gradual increase of the South component of the wind (Ditcham, Oct 1, 1908).

This classification, which though provisional is useful as a general guide, is well represented by the various lines included in the diagrams. The diagram of the South to North components is particularly noteworthy, it shows a singular symmetry of the two sides; whether from South or North, there may be a marked increase in the components in the upper layers; one of the lines starts from -5 and ends nearly at -20 and another starts from $+6{\cdot}5$ and ends at $+20$.

A typical case is illustrated by a photograph of a model (fig. 7) taken from Captain Cave's book representing the sounding which we have numbered 12. Each of its pointers represents the wind of a kilometre; its length expresses the speed; and the way it points, the direction of the wind. The gradual but marked increase in wind from the North in the upper air is very characteristic and has been noticed by many observers. The following note taken from an official communication to the Meteorological Office calls attention to it for February 25, 1918. "The wind was practically constant in direction at all heights, from 15° Its remarkable feature was its strength which increased from about 30 f/s at the ground to 160 f/s at 25,000 ft. So far as observations were obtained the wind was nearly linear, i.e. the graph of strength with height is nearly a straight line.

"Winds of this strength have now been observed at Portsmouth on three occasions (the others are 5.12.16 and 19.10.17), in each case they have been Northerly winds, their other common feature is a remarkable regularity of the wind for different observations at the same height, but at slightly different times."

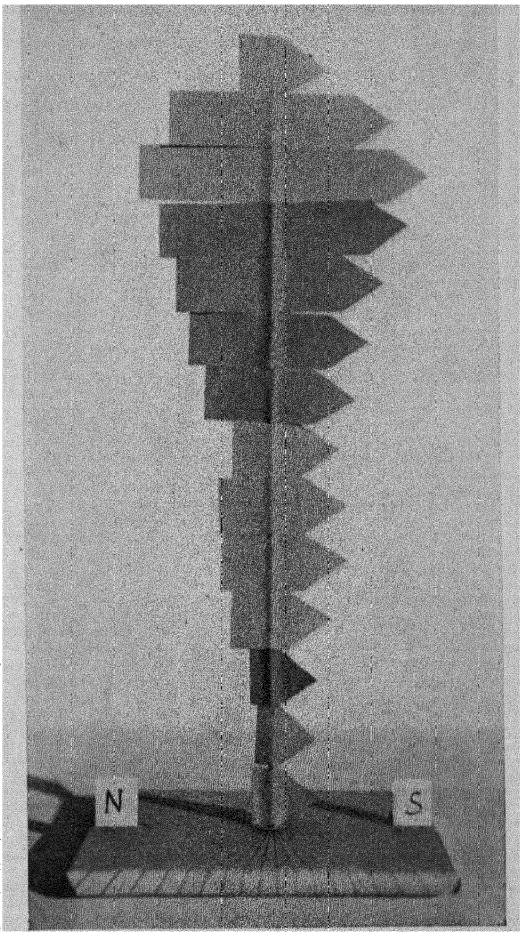
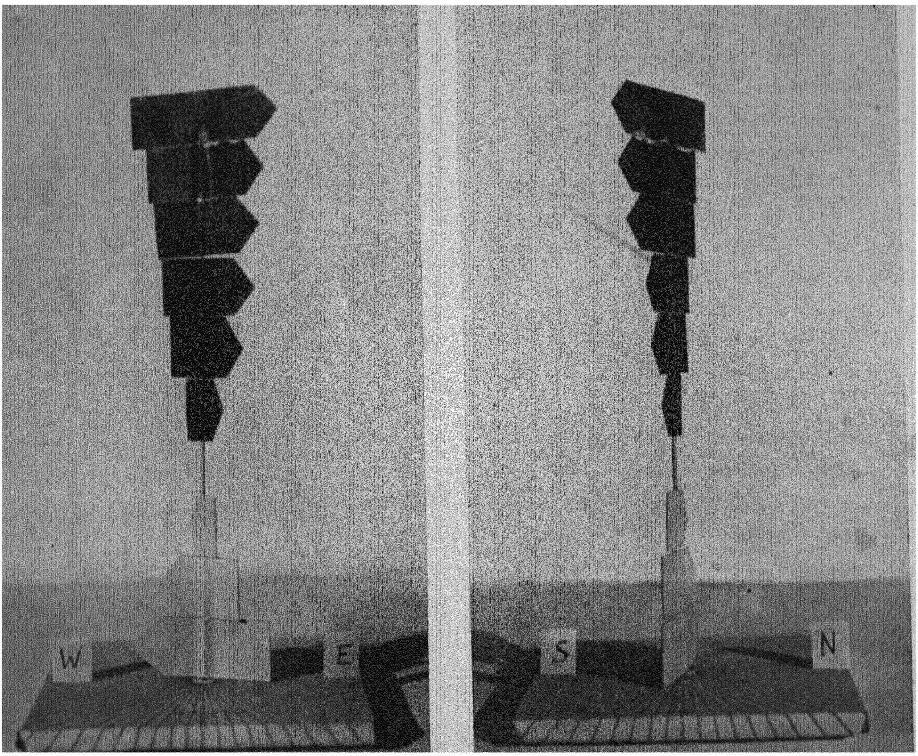


FIG. 7. Gradual increase of the North component of the wind (Ditcham, July 29, 1908).

One of the occasions mentioned in this note, namely that of October 19, 1917, has become historic because the Northerly wind increased at high levels to about 30 m/s and carried away a fleet of Zeppelins that ascended into it after attacking England and dispersed them over France where a number of them were destroyed. *Afflatus, dissipati.*

A model of the ascent numbered 16 (fig. 6), almost identical in shape, but larger in dimensions equally represents the variations in the winds of type (*b*) and shows a South wind increasing uniformly in like manner and taking on a little Westerly component in the higher levels; and just as these models

represent a gradual increase of Southerly or Northerly component of precisely similar character throughout the range from 2500 metres to 7500 metres, so another model, which is also reproduced by Captain Cave's permission, represents the sounding numbered 6 in the diagrams and shows the gradual addition of Westerly component to the original East which reverses gradually the Easterly wind in the upper layers. Hence there are three cases for increase of Northerly component, Southerly component, and Westerly component respec-



a. West to East component *b.* South to North component.

FIG. 8. Reversal of the air-current at the level of four kilometres (Ditcham, Nov. 6, 1908).

tively all on similar lines. Corresponding cases of gradual increase of Easterly wind for the same range of levels are certainly rare. But probably they do exist as Cirrus clouds are sometimes observed moving rapidly from the East.

In order that the reader may have an additional reminder of the general problem of the variation of winds in the upper air we give also Cave's model representing the rapidly increasing Westerly winds of September 1, 1907, from 5·5 m/s at the surface where its direction was actually from a point or two East of North to 13 m/s from nearly West at 2500 metres. Particulars of a more striking example are to be found in the tables[1] for the sounding of April 2,

[1] Cave, *loc. cit.* p. 88.

1907, which showed an increase from 3·5 m/s at the surface to 22 m/s at 2000 metres. These examples may be compared with those given in the diagram representing the West to East components at Aberdeen. It is instructive to speculate as to what is the dénouement of the story of which these very rapid increases with height are the beginning. So far as soundings with pilot-balloons are concerned we know that the story is generally brought to a premature conclusion by clouds or by the balloon being carried out of sight on account of distance, but there must be a development in the upper regions which is of considerable interest and may ultimately be ascertained. If the increase goes on at the same rate a velocity of the order of 100 m/s would be reached at 9000 metres. That would be beyond any of the known measurements of the velocity of clouds at any level though very high velocities were measured by means of pilot-balloons in the region of Spitsbergen. The soundings at Aberdeen, represented in fig. 4, suggest that the rapid increase of velocity is replaced by a corresponding decrease not very far up, but the mechanism of such a process is difficult to formulate. Still it must be remarked that the curves which we have put before the reader show every kind of change in every type of sounding; they differ from the extreme cases that seem almost unnatural only in degree, not in kind.

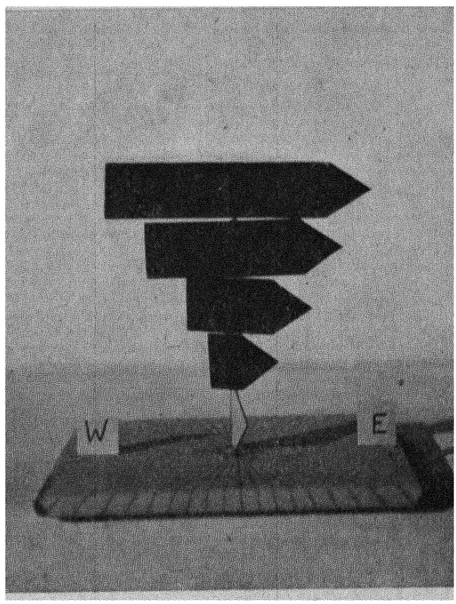


FIG. 9. Rapid increase of wind velocity with height (Ditcham, Sept. 1, 1907)

For the next stage in the presentation of what we know about the winds of the upper air we select the layer between the levels of $7\frac{1}{2}$ kilometres and $12\frac{1}{2}$ kilometres. The range of height is specially interesting because in nearly all cases it covers the transition between the troposphere and the stratosphere. For the information we rely again upon the observations recorded in Captain Cave's book. We have put together in a pair of diagrams all the observations which extended over the range of levels mentioned. A noticeable feature of these observations is that there is as a rule very little change of direction and when a change of direction is noted it is irregular and may be called wild.

We have therefore presented the diagrams showing direction and velocity for this stage instead of the two components employed for the other layers. The results are particularly noticeable because the wind-directions

group themselves very clearly about the directions NNE, SSE, SSW and NW.

Whether this grouping is fortuitous depending on the peculiar circum-

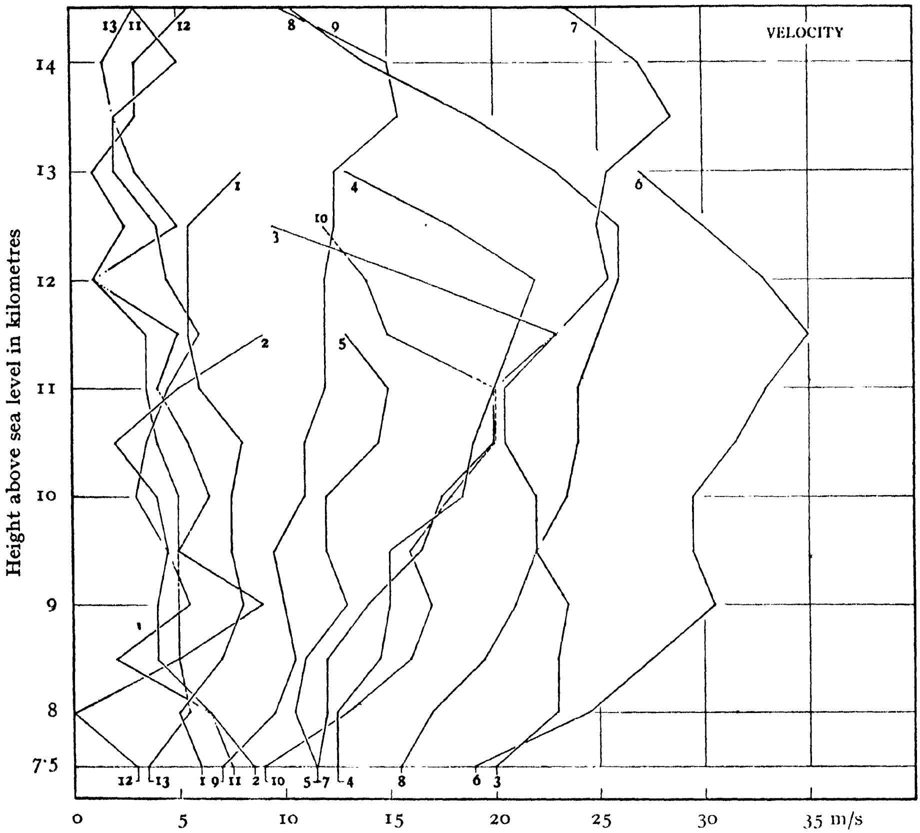


FIG. 10 *a*. Structure of the atmosphere above 7500 metres, at Ditcham Park (Cave). Velocity.

The numbers assigned to the lines in the diagrams give the dates of the observations as follows:

1.	1908	June 22	18 h. 36 m.		7.	1908	Sept. 30	16 h.	31 m.
2.	1908	June 3	10	19	8.	1908	Oct. 1	16	20
3.	1908	July 28	19		9.	1908	Oct. 2	16	20
4.	1908	July 29	19		10.	1909	May 6	18	25
5.	1908	July 30	19		11.	1909	May 7	18	29
6.	1908	July 31	19		12.	1909	Aug. 5	18	33

13. 1910 Mar. 3 16 h. 30 m.

stances when a balloon had been followed to such great heights or whether on the other hand it indicates some general property of the air-currents at those levels it is not possible to say without further investigation.

With regard to the curves representing the variations of velocity with

height, the first point to notice is that in many cases the velocities are uniformly small. About one-half of them are within 10 m/s. This is partly to be accounted for by the fact that small velocities make observations possible for great heights and partly because the winds are light, even to great heights, in many cases when the sky is free from clouds. Those winds which show an increase of velocity from the level of 7½ kilometres upwards generally carry the increase to a certain point and then show a marked decrease. The level at which the decrease commences is different on different occasions. It is at 11 kilometres on July 30, 1908, and on May 6, 1909, at 11½ kilometres on

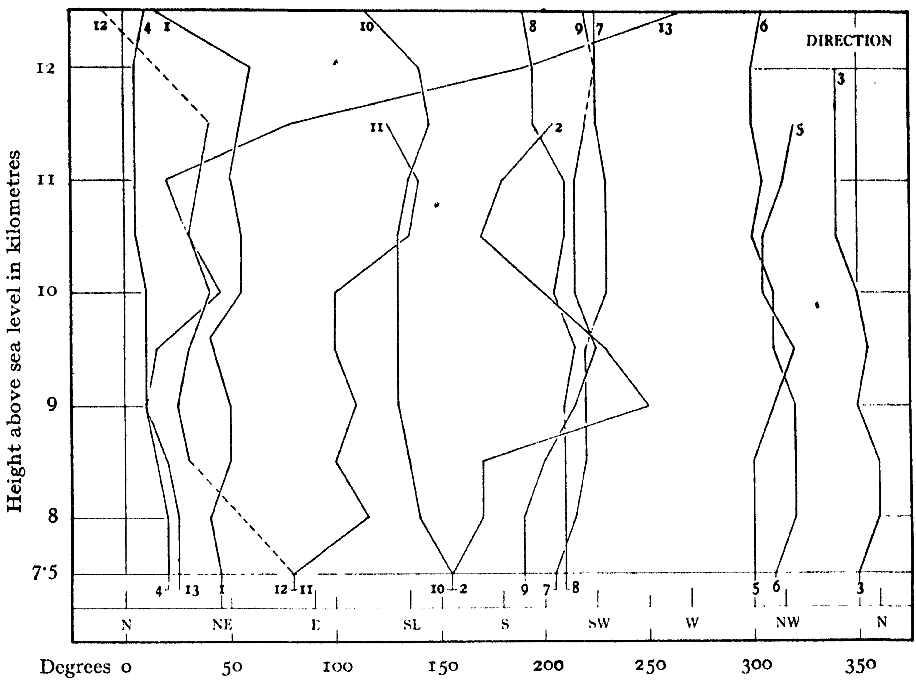


FIG. 10 *b*. Structure of the atmosphere above 7500 metres, at Ditcham Park (Cave). Direction.

July 28 and July 29 and July 31, 1908, at 12½ kilometres on Oct. 1, and at 13½ kilometres on Sept. 30, and Oct. 2, 1908. This falling off may be accounted for by a change in relation of pressure and temperature between the troposphere and stratosphere and it occurs at different heights in consequence of the variation in the height of the tropopause, to use a word which the glossary of the Meteorological Office gives for the boundary between the two. In the troposphere above the first kilometre level high pressure is associated with high temperature but in the stratosphere high pressure is marked by low temperature. A formula for the variation of wind with height which associates the falling off of the wind with the reversal of the temperature-gradient in relation to the pressure-gradient within the stratosphere is given later in the next chapter.

We have now set out the problem of the variation of wind with height in those regions of the atmosphere which are at present regularly accessible to modern aircraft and in the layer of five kilometres immediately above them. Before we pass on to consider the possibilities in respect of explanation we will add two remarks. First, we have relied for our illustrations upon discussions published before the modern development of aircraft and the corresponding development of meteorological observations designed partly, though not entirely, to aid aerial navigation. We have now a great accumulation of data obtained by observations of pilot-balloons with a single theodolite and in that respect we are much better endowed than we were. It is now possible to approach the solution of the problem of the structure of the atmosphere by rigorous statistical methods, but before that can be done with any prospect of its leading to an insight into the dynamical and physical conditions we want some guidance from general meteorological principles as to the manner in which the statistics should be classified. In face of the great variety of changes which occur a direct attack upon the whole bulk of the observations by the method of statistics is not likely to lead to satisfactory results. Meanwhile the simple inspection of the data as they accumulate has added little to, and subtracted nothing from, the complexity of the problem which is put before the reader by the examples which have been selected.

Secondly we have drawn our illustrations of individual soundings from smoothed curves which have been drawn to represent the general features of the structure of the atmosphere disclosed in the soundings by giving the values at definite intervals of height, generally at each half kilometre. The interval of 1000 feet is usual in the reports from the various Services. As the original basis of the curves there are the observations of the balloon for each minute which, if included individually, would superpose upon the diagram a series of comparatively rapid variations, sometimes large sometimes small, which are well represented by the plotting of the individual points in the diagrams appended to Captain Cave's book. These variations may be real or they may be dependent upon the incidental errors of observation of the altitude or the fluctuations in the rate of ascent of the balloon. They make the results taken from "smoothed" curves, which are drawn by eye as a reasonable account of the observations, somewhat uncertain; but, once more, whatever may be the details of the fluctuations the main features remain. We may explain by minor variations of this kind some of the irregularities shown upon a synchronous map of the results of many pilot-balloons but the fluctuations which are indicated in our diagrams are real and they call for explanation.

CHAPTER VII

The relation of the variation of winds in the upper air to the distribution of temperature

WE have reasonable ground for supposing that the winds in the upper air are closely related to the distribution of pressure; and, in turn, the variation in the distribution of pressure at different levels is dependent upon the distribution of temperature, according to the ordinary formula for the variation of pressure with height. On the principle of the first law of atmospheric motion we can put into algebraical form the relation between the changes of wind with height between successive levels, as set out in the previous chapter, and the distribution of temperature in the intervening layer.

A formula for the variation of pressure-gradient with height which in differential form may be written

$$\frac{ds}{dz} = g\rho\left(\frac{q}{\theta} - \frac{s}{p}\right) \qquad \ldots\ldots(S)$$

(where s is the horizontal pressure-gradient, q the horizontal temperature-gradient, θ and p the temperature and pressure, and g and ρ have the usual signification) was given in less conventional form in the *Journal of the Scottish Meteorological Society*[1]. The formula is deduced from the ordinary equation for the variation of pressure with height

$$\frac{dp}{dz} = -g\rho \qquad \ldots\ldots(1),$$

combined with the characteristic equation for a permanent gas

$$p/\theta = R\rho \qquad \ldots\ldots(2),$$

to these we may add the defining equations

$$s = dp/dx, \quad q = d\theta/dx \qquad \ldots\ldots(3).$$

It should be noticed that s and q have negative signs when the horizontal *gradients* of pressure and temperature are taken as *positive* in the direction of *falling pressure* and *falling temperature*.

From (3) by differentiation we have

$$\frac{ds}{dz} = \frac{d^2p}{dzdx} = -g\frac{d\rho}{dx}, \qquad \ldots\ldots(4),$$

and from equation (2) $$\frac{d\rho}{\rho} = \frac{dp}{p} - \frac{d\theta}{\theta},$$

[1] Shaw, 'Upper Air Calculus and the British Soundings during the International week, May 5–10, 1913.' *Jour. Scot. Met. Soc.*, vol. XVI, p. 167, 1913.

whence the change of pressure-gradient with height

$$\frac{ds}{dz} = g\rho\left(\frac{1}{\theta}\frac{d\theta}{dx} - \frac{1}{p}\frac{dp}{dx}\right)$$
$$= g\rho\left(\frac{q}{\theta} - \frac{s}{p}\right).$$

To obtain numerical values we substitute for ρ, $p/(R\theta)$; we get

$$\frac{ds}{dz} = \frac{g}{R}\cdot\frac{p}{\theta}\left(\frac{q}{\theta} - \frac{s}{p}\right).$$

Taking g as 981 cm/s^2 and R, for dry air, $2{\cdot}869 \times 10^6$ C.G.S. units we obtain for the equation in C.G.S. units

$$\frac{ds}{dz} = 3{\cdot}42 \times 10^{-4}\frac{p}{\theta}\left(\frac{q}{\theta} - \frac{s}{p}\right),$$

or if the variation be expressed in millibars per metre of height, and gradients in the variation over 100 kilometres, we get the rate of increase of pressure-gradient per metre of height in millibars per hundred kilometres

$$3{\cdot}42 \times 10^{-2}\frac{P}{\Theta}\left(\frac{Q}{\Theta} - \frac{S}{P}\right),$$

where Θ represents the tercentesimal temperature, P the pressure in millibars, S the horizontal pressure-gradient in millibars per hundred kilometres, and Q the horizontal temperature-gradient in tercentesimal or centigrade degrees per hundred kilometres. For our present purpose we may disregard the numerical effect of differences in R due to moisture in the atmosphere. For air saturated with moisture at 273a the constant is $2{\cdot}876 \times 10^6$ instead of $2{\cdot}869 \times 10^6$, and at 283a, $2{\cdot}884 \times 10^6$; these upper limits for the range of saturation are sufficient for the upper air and as the differences in R are quite negligible in comparison with the uncertainties in the determination of wind-velocity we may use the value of R for dry air without appreciable error.

Equation (S) was employed in the paper referred to for the purpose of explaining the dominance of the stratosphere in the distribution of pressure throughout the troposphere. The position may be set out as follows[1]. In his second report on the free atmosphere of the British Isles[2] W. H. Dines had thrown new light upon the origin of the differences of pressure at the surface by obtaining the correlation coefficient between corresponding deviations of pressure from the normal at the level of 9 kilometres and at the ground. "He had obtained results ranging from 0·67 for the last available set of a hundred soundings on the continent to 0·88 for soundings in England grouped for the winter season." Moreover the standard deviations of pressure from the normal are of the same order of magnitude at both levels. In a more recent table standard deviations for successive levels are given by Dines as follows:

Level in kilometres	8	7	6	5	4	3	2	1	0
Standard deviation in millibars ...	11·0	11·4	11·5	11·5	10·9	10·7	10·5	10·5	10·8

[1] *Proceedings of the Royal Institution*, 1916. *Nature*, vol. XCVII, p. 191 and p. 210, 1916.
[2] *Geophysical Memoirs*, No. 2. M. O. Publication, No. 210b, 1912.

That is to say at all levels within the troposphere pressure is subject to changes of the same order of magnitude in spite of the great difference in the normal values at the top as compared with the bottom. At the same time it was noted that the correlation coefficient between the deviations of pressure at the surface and of the mean temperature of the 9-kilometre column was small; in other words the mean temperature of a column of the atmosphere in the troposphere, between the surface and the level of 9 kilometres, has little to do with the general distribution of pressure over the country. Its effects may be regarded as occasional and local.

This aspect of the subject, which is of special interest in connexion with the explanation of the general circulation of the atmosphere, was first referred to in the official preface to W. H. Dines's report, and the equation which we are now considering affords a satisfactory explanation of the position. It is certain that the distribution of pressure at the upper levels, of which that at 9 kilometres is taken as typical, is transmitted to the surface and defines the distribution there, subject to any variations caused by the distribution of density of the air of the troposphere, between the level of 9 kilometres and the surface, in consequence of the variations of pressure and temperature. The equation (S) demonstrates that these variations are not likely to be large for the region of the British Isles because according to Dines's results Q and S are of the same sign throughout the troposphere except in the lowest layer of one kilometre and $Q/\Theta - S/P$ will depend upon the numerical difference between the two ratios. Between ground-level and 9 kilometres Θ may be said to vary from about 280a to 220a while P varies from 1010 mb. to 300 mb.; thus the denominators of the two terms vary very differently within the range of levels which has been specified; that for pressure is reduced roughly to one-third of its sea-level value, while that for temperature is only reduced to three-quarters of its sea-level value. Consequently, between the 9-kilometre level and the surface, S/P runs through a considerable range of values on account of the variation of the denominator alone whereas Q/Θ remains comparatively steady. Hence for a considerable range of values of Θ and P it is likely that somewhere between 9 kilometres and the surface the difference $Q/\Theta - S/P$ will be zero. In that case there will be positive values in the lower region and negative values in the upper region: and the effect of the whole troposphere will be small.

This accords with our knowledge of the régime of winds. If the pressure-gradient remained constant throughout the vertical height from the surface to the level of 9 kilometres the product $V\rho$ would be invariable; the velocity of the wind would increase in inverse proportion with the change of density so that the mass of air passing would be the same at all levels as Egnell[1] and Clayton[2] were led to suggest from observations of clouds. We have seen that the actual wind shows almost every kind of variation with height but on the

[1] *Comptes Rendus*, vol. CXXXVI, p. 358, 1903, 'International Cloud Operations,' Trappes, 1896–7.

[2] McAdie, *The Principles of Aerography* (Rand, McNally & Co.), p. 81.

average perhaps less increase in the upper layers than that corresponding with a uniform gradient.

The variations of gradient between 2·5 kilometres and 7·5 kilometres may be regarded as depending mainly on the distribution of temperature because the distribution of pressure is directly related to the distribution of temperature. Hence we may regard the distribution of wind in the vertical as ordinarily controlled by the distribution of temperature, allowance being made for those cases in which the winds are affected locally by thermal convection. Such cases are not likely to be disclosed by observations with pilot-balloons because the thermal convection which produces the disturbance is likely to cause cloud and terminate the sounding.

From the equations already given combined with the equation for the geostrophic wind,

$$v\rho = s/(2\omega \sin \phi) \qquad \text{......}(G),$$

a formula can be obtained for the variation of wind with height.

Thus, from equation (G), taking v the component of velocity parallel to the y-axis drawn northward,

$$\begin{aligned}
\frac{1}{v}\frac{dv}{dz} &= \frac{1}{s}\frac{ds}{dz} - \frac{1}{\rho}\frac{d\rho}{dz} \\
&= -\frac{g}{s}\frac{d\rho}{dx} - \frac{1}{\rho}\frac{d\rho}{dz} \\
&= -\frac{g\rho}{s}\left(\frac{1}{p}\frac{dp}{dx} - \frac{1}{\theta}\frac{d\theta}{dx}\right) - \left(\frac{1}{p}\frac{dp}{dz} - \frac{1}{\theta}\frac{d\theta}{dz}\right) \\
&= -\frac{g\rho}{s}\left(\frac{s}{p} - \frac{q}{\theta}\right) + \frac{g\rho}{p} + \frac{1}{\theta}\frac{d\theta}{dz} \\
&= \frac{1}{\theta}\left(\frac{g\rho q}{s} + \frac{d\theta}{dz}\right), \\
\frac{s}{v}\frac{dv}{dz} &= \frac{1}{\theta}\left(g\rho q + s\frac{d\theta}{dz}\right).
\end{aligned}$$

But

$$s/v = \rho \times 2\omega \sin \phi,$$

whence

$$\begin{aligned}
\frac{dv}{dz} &= \frac{1}{\rho\theta \times 2\omega \sin \phi}\left(-q\frac{dp}{dz} + s\frac{d\theta}{dz}\right) \\
&= \frac{1}{\rho\theta \times 2\omega \sin \phi}\left(\frac{dp}{dx}\cdot\frac{d\theta}{dz} - \frac{d\theta}{dx}\cdot\frac{dp}{dz}\right) \qquad \text{......}(5a).
\end{aligned}$$

The equation has been put also into other equivalent forms as

$$\frac{dv}{dz} = \frac{v}{\theta}\frac{d\theta}{dz} + \frac{g}{2\omega \sin \phi}\frac{q}{\theta} \qquad \text{......}(5b),$$

and

$$\frac{1}{v}\frac{dv}{dz} = \frac{1}{\theta}\left(\frac{d\theta}{dz} + \frac{gp}{R\theta}\frac{q}{s}\right) \qquad \text{......}(5c).$$

The corresponding equation for a wind along the x-axis will be

$$\frac{du}{dz} = -\frac{1}{\rho\theta \times 2\omega \sin \phi}\left(\frac{dp}{dy}\cdot\frac{d\theta}{dz} - \frac{d\theta}{dy}\cdot\frac{dp}{dz}\right),$$

because *if the pressure increases to the northward* with increasing y the corresponding change of wind will be from the *east*. We have referred the formula to an x-axis drawn eastward and a y-axis drawn northward because this resolution into components is required for computing the variation of direction with height. The equations may be taken as applicable to any direction of the wind if the y-axis be taken at right angles to the run of the wind and drawn to the left.

The several forms of equation cannot be applied generally to the numerical evaluation of special cases of the variation of wind with height in the free air because the individual values of the horizontal gradients q and s and the lapse-rates ($d\theta/dz$ and dp/dz) of temperature and pressure are not known. We could compute the horizontal gradient of pressure from the wind and the lapse-rate of pressure can be taken from normal values without any serious error. So also can the value of ρ because that depends upon the ratio p/θ which shows as a rule very small variations from the normal for the month; they seldom exceed five per cent. and are generally much less: often within one per cent.[1]

For the layers near the surface we have observations of temperature at the ground-level from which we can form an estimate of the horizontal gradient that may help us to deal with the relation of wind to the distribution of temperature at moderate heights and this part of the subject will be treated in a subsequent chapter.

And for the free air we can use the observations of variation of wind with height obtained by means of pilot-balloons to compute the values of q at successive levels with the understanding as to using normal values of θ and p/θ. Before doing so we must note an interesting application of equation (5) made by W. H. Dines[2].

He has shown that on the basis of the first law of atmospheric motion there will be no variation of wind with height if the isobaric surfaces are also isothermal. The demonstration follows directly from equation (5) because the conditions prescribed may be expressed in the form that the variation of pressure in any direction is proportional to the variation of temperature so that

$$\frac{dp}{dx} : \frac{dp}{dy} : \frac{dp}{dz} :: \frac{d\theta}{dx} : \frac{d\theta}{dy} : \frac{d\theta}{dz},$$

and in that case the quantity within the bracket of equation (5) becomes zero. It should be noted that this condition for no variation of wind with height is satisfied where the atmosphere is uniformly isothermal. Mr Dines's demonstration is as follows:

Let $ABCD$ be a vertical section at right angles to the gradient wind, AB and CD being sections of the isobaric surfaces, and AC and BD vertical straight lines (fig. 1). If v be the gradient wind—i.e. the wind at right angles to the paper—then the tangent of the slope of AB is

$$2\omega v \sin\phi + v^2/r : g,$$

for $2\omega v \sin\phi + v^2/r$ is the horizontal acceleration and g the vertical. Similarly,

[1] See Part I, chap. II.

[2] *Nature*, vol. XCIX, p. 24, 1917.

the slope of CD is $2\omega V \sin \phi + V^2/r : g$. If, then, v is greater than V, BD must be greater than AC. Now the pressure-difference between A and C is equal to the pressure-difference between B and D, since AB and CD are isobaric lines; and since the corresponding elements in the two air columns AC and BD are of equal pressure, and the density in BD less, the temperature in BD must be higher than that in AC. That is, if v be greater than V, then t is greater than T. Or if v is the same as V, t must be equal to T, and the lines AB, CD are isothermal as well as isobaric.

An important conclusion of a very general character follows immediately from this proposition. If the isobaric surfaces are also isothermal a vertical cross section of the atmosphere will show lines of equal pressure and equal temperature having the same slope in the region where there is no variation of wind with height. In any part of the section where the slope of the isothermal line is steeper than that of the isobaric line the wind will increase with height and where, on the other hand, the isobaric line is steeper than the isothermal line the velocity of the wind will diminish with height. By

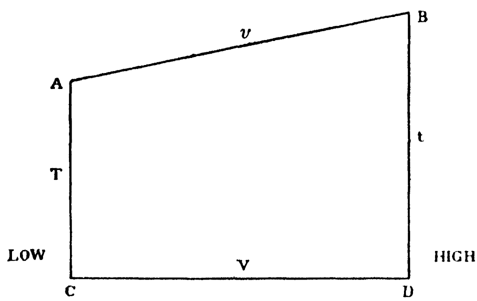


FIG. 1.

grouping together a large number of observations of pressure and temperature at all heights up to 20 kilometres W. H. Dines[1] has constructed a diagram representing the mean distribution of temperature in relation to pressure at different levels in the upper air, from which it appears that the isothermal lines in a vertical section reach a maximum height in the highest pressure and a minimum in the lowest pressure. In these regions the isothermal lines are parallel to the isobaric lines and there is no variation of wind with height, a conclusion which is supported by observation so far as the facts go. Elsewhere the isobaric lines are distinctly more nearly horizontal than the isothermal lines. Hence it follows that in a region between high pressure and low pressure the wind in the successive layers of the troposphere should in normal circumstances show an increase of velocity with height.

We may assign a numerical estimate of the application of this proposition by using the average results for pressures, temperatures and densities at different heights, taken from Dines's diagram, as given in the *Meteorological Glossary*[2] and assuming the differences of pressure there given to be

[1] *Phil. Trans.*, vol. CCXI, A, p. 253, 1911. The diagram in another form is reproduced in *Nature*, vol. XCIX, p. 24, 1917, and *Proc. Roy. Inst.*, March 10, 1916.

[2] M. O. Publication, No. 225 ii, s.v. Density.

distributed over a horizontal stretch of 1000 kilometres which would represent a pressure-gradient of 5 mb. per hundred kilometres. Substituting the values thus obtained in formula (5 *a*) of this chapter we get numerical values of the increase of wind-velocity with height under the prescribed conditions as follows:

NORMAL INCREASE OF WIND-VELOCITY AT DIFFERENT HEIGHTS UNDER THE NORMAL CONDITIONS OF TEMPERATURE FOR A SURFACE-GRADIENT OF 5 MB. PER 100 K.

Height in kilometres	1	2	3	4	5	6	7	8
Normal increase of velocity per kilometre in metres per second	0·7	1·8	2·0	2·8	3·1	3·4	3·8	3·6

The application of the equations to the approximate evaluation of the horizontal gradient of pressure and temperature, assuming uniformity of change over a whole kilometre, is given in *Principia Atmospherica*[1] and further developed in a paper before the Royal Meteorological Society[2]. Tables for facilitating the calculations are given in the sect. II of the *Computer's Handbook*[3], § 3.

The calculation proceeds from the formula,

change of pressure difference for 1 kilometre, $\Delta s = 34{\cdot}2\,\dfrac{P}{\Theta}\left(\dfrac{\Delta\Theta}{\Theta} - \dfrac{\Delta P}{P}\right)$,

where $\Delta\Theta$ is the change of temperature per hundred kilometres and ΔP the corresponding change of pressure: and for the wind-velocity at any level

$$V = \frac{R}{2\omega\sin\phi}\,\frac{\Theta}{P}\,\Delta P.$$

Taking the components U, from W. to E., and V from S. to N. separately, we get for the components of pressure-difference at any level

$$\Delta_{\mathrm{N}}P = \frac{1}{K}\,\frac{P}{\Theta}\,U,$$

and

$$\Delta_{\mathrm{W}}P = \frac{1}{K}\,\frac{P}{\Theta}\,V,$$

where K represents $R/(2\omega\sin\phi)$, and for the components of temperature-difference

$$\Delta_{\mathrm{N}}\Theta = \frac{\Theta}{P}\left(\frac{\Delta s_{\mathrm{N}}}{34{\cdot}2}\times\Theta + \Delta_{\mathrm{N}}P\right),$$

$$\Delta_{\mathrm{W}}\Theta = \frac{\Theta}{P}\left(\frac{\Delta s_{\mathrm{W}}}{34{\cdot}2}\times\Theta + \Delta_{\mathrm{W}}P\right).$$

Θ/P and Θ are taken from tables and the calculation is applied to the change of wind-velocity in successive kilometres. It is only properly applicable when the rate of variation of wind-velocity is uniform over the range, and the precise point at which the computed value of the horizontal temperature-gradient is

[1] Shaw, *Proceedings Royal Society of Edinburgh*, vol. XXXIV, p. 77, 1914.

[2] Shaw, 'The Interpretation of the results of Soundings with Pilot-balloons,' *Q. J. Roy. Met. Soc.*, vol. XL, p. 112, 1914.

[3] M. O. Publication No. 223.

operative is rather doubtful. Still the computations throw a considerable amount of light upon the way in which different distributions of temperature affect the structure of the atmosphere and the general conclusions to be drawn from them are not unreasonable.

Six cases are given in detail in the paper on the interpretation of the results of soundings with pilot-balloons. They deal with the soundings which were selected by Cave for illustration in his book by photographs of models. The details of the computation of an additional case representing an upper wind from North West crossing a lower wind from South West are given in *Principia Atmospherica*, and also in the *Computer's Handbook*.

The final result of the computation is to enable us to calculate the distance between consecutive isobars and consecutive isotherms at the several levels and to determine also the direction of the isobar and the isotherm. Hence we can draw for each level the positions of two consecutive isobars and two consecutive isotherms which give us an index of the distribution of pressure and temperature at the several levels which are in accord with the observed changes in the wind.

The results for four cases are given in the diagrams of figure 2.

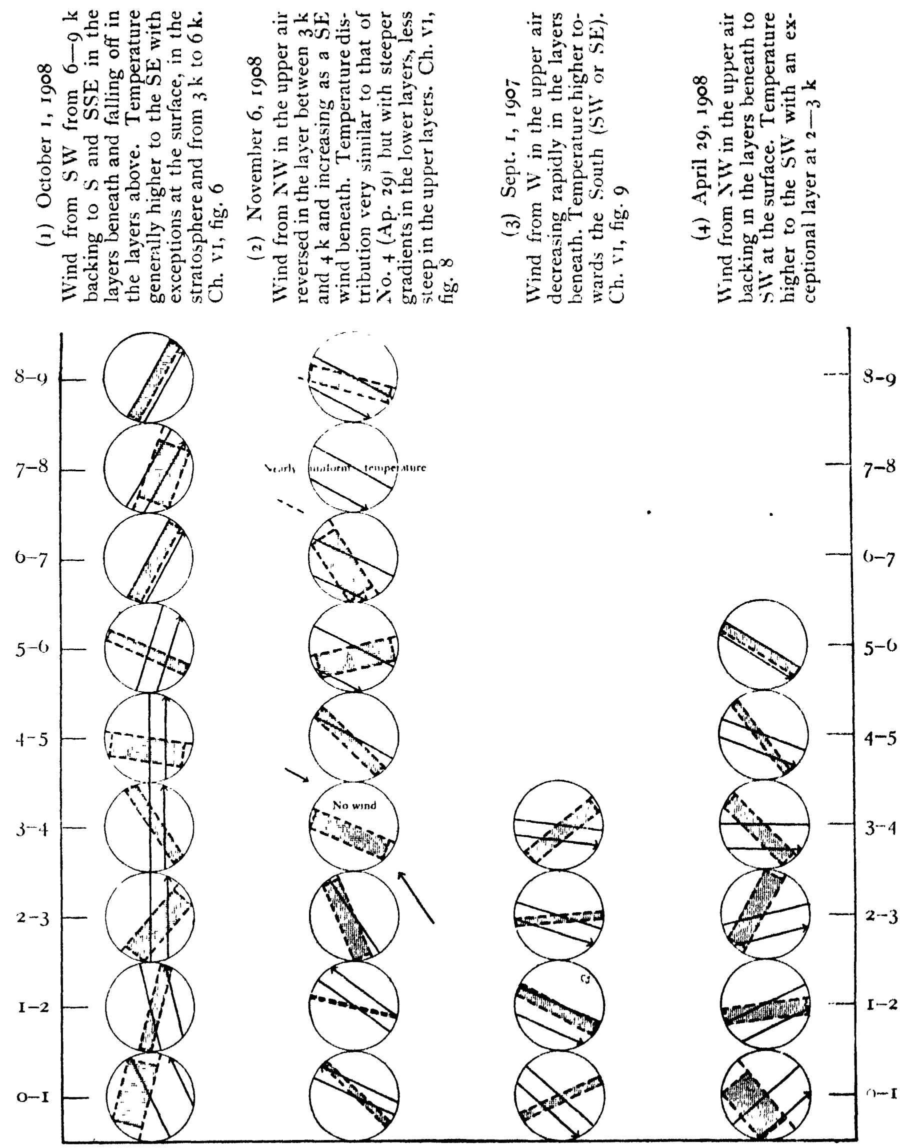
From these diagrams we may draw certain inferences. On September 1, 1907, when starting from the surface-wind of 5·5 m/s from 25° East of North there was a backing to a wind of 4·5 m/s from 290° at 1 kilometre and thereafter a rapid increase combined with backing until the wind was 16 m/s from 270° at 4 kilometres; the air was always colder towards the North, but the temperature-gradient points towards the West of North except between one and two kilometres when the gradient was towards the North East. (Chap. VI, fig. 9.)

On April 29, 1908, when the wind gradually changed from a light wind from South West at the surface to a strong wind from North West at 6 kilometres, the temperature-gradient was towards the North East except between one and three kilometres and generally the fall of temperature to the North East was rapid. The isotherms gradually drew from being across the wind at the surface to being nearly parallel to the wind between five and six kilometres.

On October 1, 1908, when the wind changed from SSE at the surface to SW with regularly increasing strength from eight kilometres upwards the gradient of temperature was generally towards the West with a good deal of variation in direction between SW and NW until the sixth kilometre was reached when it ranged itself according to the pressure-gradient. The two steps four kilometres to five kilometres and five kilometres to six kilometres, show the isotherms across the isobars with the wind blowing directly towards the colder region. (Chap. VI, fig. 6.)

On November 6, 1908, when the wind changed from South East in the lowest layer by gradual diminution through calm, between three and four kilometres, to North West with regular increase in the upper layers, the temperature-gradient was generally towards the North East with a notable

FIG. 2. DIAGRAMS illustrating the relation of changes in wind with height to the distribution of temperature in successive layers. Separation of consecutive isobars and isotherms computed from the observations represented in figs. 6, 8, and 9 of chap. VI, with an example of a NW wind in the upper air crossing a SW wind at the surface.



Within each circle is a plan of consecutive isobars and isotherms for the layer indicated in kilometres by the figures on either side of the diagrams. In each plan the pair of full lines represent the isobar which passes through the point of observation and the isobar for a pressure of 1 mb. higher. The pair of dotted lines represent the isotherm which passes through the point of observation and the isotherm of the next higher degree on the centigrade or tercentesimal (absolute) scale. The radius of each circle represents 250 kilometres.

exception between five and six kilometres when the isotherms crossed the wind and with one exception also between two and three kilometres when the temperature-gradient pointed nearly West. (Chap. VI, fig. 8.)

The study of these diagrams suggests to us the desirability of regarding the influence of the distribution of temperature as affecting the transmission of pressure downwards from above, not as we are accustomed to think of it as building up the pressure-distribution in the upper air, because we recognise that when we have got to the top of our building, the differences to be accounted for are just as great as they were at the start. We should look upon the distribution of pressure at the surface as being modified in its transmission from above by the temperature of the air, the motion of which has to be controlled and maintained by the distribution of pressure at the successive layers.

In accordance with a suggestion of Lieut.-Col. Gold[1] we may consider the general principle that the wind at the top of any layer differs from the wind at the bottom of the layer by a vector component depending upon the distribution of temperature within the layer which we may call the thermal wind. Thus the top wind is the geometrical sum of the bottom wind and the thermal wind for the layer. The student may build up a working idea of the variation of wind with height by habitually forming an estimate of the thermal wind when observations of temperature are available; or, *vice versa*, by noting the difference between an upper wind and a lower wind he may obtain the thermal wind and hence a working idea of the thermal structure of the layer. But this method requires the bottom wind to be corrected in the ratio of top temperature to bottom temperature.

If on the occasions of the soundings with pilot-balloons which are represented in these diagrams we had been fortunate enough to have observations of temperatures at a sufficient number of stations to give a trustworthy measure of the temperature-gradient in the immediate locality we might have subjected the conclusions to the direct test of observation. We have not yet reached that favourable position. Observations for this purpose require a high degree of precision. The temperature-gradients indicated seldom amount to more than one degree in a hundred kilometres and observations at different stations by means of ballon-sondes do not claim higher accuracy than a degree, so that temperature-gradients obtained from the few stations in the British Isles which are occasionally available can hardly be regarded as final evidence of the horizontal-gradient of temperature for levels within the troposphere where the isothermal surfaces are very nearly horizontal. But the direct comparison would certainly be interesting and it is much to be regretted that the occasions of simultaneous observations with ballon-sondes and pilot-balloons are so few.

We may however consider the results obtained by the calculations in the light of our knowledge from other sources bearing upon the question. We have already mentioned that for levels above four kilometres W. H. Dines has shown that there is very high correlation between the deviations from the normals of pressure and temperature obtained by the soundings with ballon-

[1] M. O. Correspondence, 14117, 1918.

sondes. That relation would be explained if the temperature-gradient at each level were always along the line of the pressure-gradient and proportional thereto, that is to say if the isotherms were parallel to the isobars and at a proportionate distance apart. It is therefore interesting to note that in all the diagrams there is a very definite tendency for the isotherms to become parallel to the isobars especially in the upper levels. There are only two examples of a temperature-gradient nearly opposite to the pressure-gradient, those are between two and three kilometres on April 29, 1908, and between one and two kilometres on November 6. But there are a number of cases in which according to the computation the isotherms are at right angles to the isobars. These may be due to errors in the data upon which the computations are based, but if they are real they may be useful in explaining atmospheric processes. It was pointed out in *Principia Atmospherica*, for example, in discussing the case of April 29, 1908, when a surface-wind from South West was passing under an upper wind from North West, that the South West wind had its temperature-gradient to the North East and the current as it went forward was continually replaced by warmer air which passed under a layer in which there was no corresponding change of temperature; that state of things must ultimately result in instability which is characteristic of the South-Westerly wind of an advancing depression, and the process, which would eventuate in rainfall, is inevitable if in consequence of the distribution of temperature beneath it the gradient for North-Westerly wind is transformed into a gradient for South-Westerly wind near to the surface. This case happens not infrequently when a low pressure system is passing away to the North Eastward and is followed by another depression. The South-Westerly wind of the coming depression appears first at the surface while the North-Westerly wind remains at the higher levels.

On the other hand the cases of isotherms transverse to isobars in the upper air may be indications of local variations of temperature that are the result rather than the cause of convection, or local inversions of the lapse of temperature due to pressure changes in the stratosphere.

An example of a complete reversal of wind-velocity with height which may be accounted for by an inversion of lapse of temperature is given in *Principia Atmospherica* from a sounding by J. S. Dines at Pyrton Hill on October 16, 1913. "On that day there was a sudden change of wind between 1100 and 1500 metres height from a fairly steady wind from nearly due south into one almost as steady from due north, the change being accomplished within half a kilometre. For the layer between 500 and 1100 metres the analysis in this case shows a temperature distribution in isotherms nearly north and south with the warmer air in the east and above 1500 metres an entirely different distribution with isotherms nearly east and west and cold to the northward. The intermediate layer 400 metres thick showed a very rapid increase of temperature to the west as much as 7° C. per hundred kilometres.

"The complete arrest of the upper northerly current and production of a calm by the annihilation of the gradient between 1500 metres and 1100

metres is very remarkable but nevertheless a real fact. The accompanying temperature difference is probably due to a strong 'inversion' at a height of about 1500 metres at the place of observation and at about 1100 metres at a place 100 kilometres distant to the west. On that occasion it lasted for some time, as it was found an hour afterwards by a second balloon; but it must be remembered that it was a region of no velocity and therefore the warm and cold airs at those levels were not moving."

Such a distribution was by no means improbable on the day of the observation. The land area of England had been covered by cold air above which there was probably an inversion as there is above a fog. The morning observations for 7 a.m. showed a temperature of 38° at Nottingham with fog, and of 39° at Bath with blue sky. While London had a temperature of 50°, Yarmouth 50° and Pembroke 47°. The wind at Nottingham at that hour was from WSW but NE at Bath and ENE at Pembroke.

The conclusion that in the upper air the isotherms are generally parallel to the isobars and the gradients of pressure and temperature at the several levels proportional is interesting from the bearing that it has upon Egnell's or Clayton's law of the variation of wind with height. We have seen that the law requires that the pressure-gradients should be the same at all heights and from equation (S) the condition becomes

$$d\theta/\theta = dp/p.$$

We have seen that on occasions the wind shows continuous increase with height from the North, from the South and from the West, very rarely from the East. We may expect the law to be verified therefore by winds from South, West or North; if it is also verified for winds from the East it would confirm a conclusion at which W. H. Dines had arrived on other grounds that there are no preferences for direction of temperature-gradients in the upper air. In the lower layers the circumstances under which temperature increases to the North (the condition required for the maintenance of the pressure-gradient in an Easterly wind) are very rare and peculiar but in the upper air the rule of proportionality of temperature-difference to pressure-difference is quite general and an Easterly wind increasing with height ought not to be regarded as out of the question at those levels.

The direct relation of the gradient of pressure to the gradient of temperature which is normal in the troposphere is reversed in the stratosphere, that is to say high pressures are cold in that region as compared with low pressures. In that case Q/Θ takes the negative sign when S/P is positive; and, in consequence, the gradient of pressure and the wind-velocity must fall off with height. We have noticed that there is a tendency for the wind-velocity to fall off in the stratosphere. The diagram in fig. 10 *a*, p. 76, shows six examples. From the measurements of the change of wind-velocity the gradient of temperature just above the base of the stratosphere has been computed by formula (5 *b*) as shown in the following table[1]:

[1] *Proc. Roy. Inst.* 1916, March 10.

Date 1908	Rate of change of velocity in the stratosphere m/s per k.	Horizontal temperature-gradient Computed. Degrees per 100 k.	Horizontal temperature-gradient Observed. Degrees per 100 k.
July 27	—	—	2·5
,, 28	−13	4·0	—
,, 29	−11	3·3	3·3
,, 31	− 5	1·5	—
Oct. 1	− 7	2·1	—

The days in July belonged to the international week upon which soundings were made with ballon-sondes at three stations in England, one in Scotland and one in Ireland. For July 27 and 29 enough balloons were found and returned to give the material for constructing the models which are represented in chap. II of part I, and from the models the temperature-gradient can be determined with some confidence because in the stratosphere the isothermal surfaces are more nearly vertical than horizontal. The horizontal temperature-gradient as determined by measurements of the model is 3·3a per 100 kilometres which agrees exactly with the value computed from the change in wind-velocity. The exactness of the agreement is doubtless fortuitous but it is interesting to see that results of the same order of magnitude are given for the other days for the values computed from the change of velocity and on the only other occasion available for the observed value of the horizontal gradient of temperature.

If the falling off of velocity in the stratosphere follows the distribution of temperature, as it apparently does in fact, it would also follow that the wind-velocity and with it the pressure-gradient would become zero within a few kilometres. Looking at the diagram of fig. 10*a*, chap. VI, it would appear that zero velocity and therefore zero gradient would be reached at 13 kilometres on July 28, 1908, at 14 kilometres on July 29, 1908, and May 6, 1909, at 15 kilometres on May 7, 1909, at 16·5 kilometres on Sept. 30, 1908, and at 17·5 kilometres on July 31, 1908. If we suppose the temperatures in the vertical to remain uniform beyond these limits, as temperature is usually nearly uniform in the vertical within the stratosphere, we must conclude that the pressure in the warm column over the "low" will diminish more slowly than that in the cold column over the "high." Hence above the level of no gradient and no wind there will be a new region in which the high is warmer than the low and the wind will be reversed in direction and gradually increase in magnitude as the heights increase until some change takes place in the distribution of temperature. Thus at great heights in the stratosphere an increasing Easterly wind may be found above a Westerly wind in the troposphere and so on for the other directions. Cave[1] has noted an interesting case in which a Southerly wind began to show at 48,000 ft. increasing to 44 miles per hour from the same direction at 58,000 ft.

It may here be recalled that as a result of the inquiry into the phenomena due to the eruption of Krakatoa in August, 1883, an Easterly wind of about

[1] Cave, M. O. Correspondence, 4158, 1918.

80 miles an hour was identified in the equatorial regions[1]. Very high velocities for very high levels are sometimes indicated by the luminous trails of meteors[2].

From the examples which have been given it will be seen that the hypothesis of an atmosphere in which the wind-velocity is normally adjusted to balance the pressure-distribution enables us to explain many of the ascertained facts that have been disclosed by observations of the upper air. Among them we may recall the conditions for change of wind-velocity with height, the general arrangement of pressure-distribution according to temperature-distribution in the upper layers of the troposphere, the falling off of wind in the stratosphere and the change of wind over an "inversion." It also justifies us in regarding the stratosphere as the dominant region of the atmosphere so far as the distribution of pressure is concerned. The tendency of meteorological study in the past has been to regard the structure of the atmosphere as built upon the foundation which we see laid out at the surface. We shall probably find fewer difficulties in the path of the study if we regard the pressure-distribution at the surface as controlled by the stratosphere and only modified locally by the irregularities of temperature that are to be found in the lower layers. The proportionality of changes of pressure to changes of temperature in the section from four to eight kilometres is probably of the highest significance for the comprehension of the structure of the atmosphere.

[1] 'The Eruption of Krakatoa and Subsequent Phenomena.' *Report of the Krakatoa Committee of the Royal Society*, 1888, pp. 325–333.

[2] See an interesting article on the travel of Meteors in *Chambers' Encyclopedia*, s v.

CHAPTER VIII

Wind at the 500-metre level and pressure-distribution at the sea level

For many years it has been the practice of the Meteorological Office to notify the direction and velocity of the geostrophic wind at the surface as the best available estimate of the wind at the level of 500 metres or 1500 feet. Subsequently the result of the computation was notified in slightly different form as the probable wind at from 1500 to 2000 feet. The practice originated with a request from the Ordnance Committee for an estimate of the direction and velocity of the wind in the upper air for experimental work at Shoeburyness. The request came about the year 1905 just at the time when Gold had in hand the material for his report on Barometric Gradient and Wind Force, and had found practical agreement on the average between the wind observed by means of kites at the 500-metre level and the gradient wind at the surface for Oxshott and for Lindenberg.

From that time onward times of experimental firing were notified and an estimate made in the Office of the geostrophic wind with allowance for the difference between the time of the maps and the time of firing. Either by accident or design, because wind is only a disturbing element in gunnery, the times selected for the experiments were generally marked by the absence of any notable wind or gradient at the surface, and in consequence there was little guidance on the maps for an estimate of the wind in the upper levels. After some years of practice word came that the estimates of wind in the upper air were no longer required. No reason was given, but there is ground for believing that the Ordnance Committee came to the conclusion that the method of determining the wind-velocity by the gradient had been tried and found to be not more satisfactory than guessing the upper wind by the traditional practice of applying a formula to the wind observed at the surface.

It is a striking example of the manner in which scientific inquiry should not be conducted because there was no exchange of views or of experience between those who asked the question and those who gave the answer. The question whether in ordinary circumstances the surface-gradient is or is not a guide to the wind at the level of 500 metres or thereabouts remained exactly where it was, and a good deal of time was spent to no useful purpose because the opportunity for using the scientific method of checking calculations by results and examining the occasions of serious discrepancy was lost.

The observations with pilot-balloons, which have been set on foot at many stations since the inquiry referred to, enable us to take up the question again in much more favourable conditions, and the inquiry is still necessary because with all the experience of the intervening ten years the geostrophic

velocity is still in ordinary circumstances the best suggestion that the Meteorological Office has to offer for the wind at 1500 to 2000 feet for any locality where a direct observation of a pilot-balloon with two theodolites or its equivalent is not available.

We propose accordingly to give some attention to this particular question. Many desultory comparisons of the wind at 1000 ft. or 2000 ft., or at 500 metres with the geostrophic wind at sea level have been made in the Office with the general conclusion that while the agreement is good on the whole, it is better if the mean gradient over a considerable area is estimated and the means of a number of observations with pilot-balloons at the selected levels are taken for the comparison. The difference is illustrated by the following figures supplied by J. S. Dines and E. V. Newnham:

Correlation between surface-gradient wind and observed wind at	2000 ft.	at 3000 ft.
For single station (E. V. N.) (72 observations in the winter of 1916–17)	0·67	0·72
For three or more stations (J. S. D.)	0·76	0·82

The mean ratios[1] of the winds, at the levels specified, to the geostrophic winds of the surface isobars in Newnham's comparison is ·90 in each case; an allowance for the curvature of the isobars would make the ratio still nearer to unity. But even with this modification there are occasions on which the observed winds are not in agreement with the computed winds. To furnish a reply to the definite question whether the winds as observed by pilot-balloons at certain meteorological stations near the coast were in accord with the gradient, Captain Cave made an investigation for six stations of which one is on the East coast, one on the South coast of England, two are near the East coast of Scotland and two near the coast of North East England. The results are as follows:

Comparison of the winds observed at 1000 ft. and 2000 ft. with the geostrophic wind at sea level: mean results for six stations on the East of Great Britain for January 1917.

Direction. Deviation from the geostrophic wind in "points."

	Backing >4	3 or 4	1 or 2	Less than 1	1 or 2	3 or 4	Veering >4
Percentage frequency of observations at 1000 ft.	4·5	24	36	13	14·5	·6	2
Percentage frequency of observations at 2000 ft.	1·5	12	39	19	18	7	3·5

There is an obvious mode for deviation in the direction of backing of the actual wind through one or two points which is more pronounced at 2000 ft. than at 1000 ft.

[1] It should be noted that the correlation ratio between two magnitudes is quite distinct from the ratio between their mean values which may be large or small even when the correlation ratio for deviations from the mean values is equal to unity. The difference is well exemplified in the case of the relation of the surface-wind to the geostrophic wind for which W. H. Dines gives the correlation ratio ·75 based upon 200 observations at Southport and 200 at Alnwick; at the former place the ratio of the mean values is only ·44 and at the latter probably not greater than ·33.

Velocity. Ratio of the observed wind to the geostrophic wind at sea level.

	<·7	·71 to ·80	·81 to ·90	·91 to 1·0 1·0 to 1·10	1·11 to 1·20	1·21 to 1·30	>1·3
Percentage frequency of observations at 1000 ft.	29	18	14	18	8	4	9
Percentage frequency of observations at 2000 ft.	14	11	15·5	26	8	4	21·5

The modes in this case are less pronounced than they are in the case of direction and the high values for the frequency of ratios less than ·7 at 1000 ft. and greater than 1·3 at 2000 ft. show that the magnitudes included in the comparison were not in strictly comparable form.

Considering that we are dealing with winds in the free air the agreement as regards direction is not good and the irregularities as regards velocity are so notable that no generalisation is possible. The results for individual stations are no more satisfactory than the means for the six.

The results draw attention to the fact that there are special causes of irregularity at stations near the coast and the selection of these stations might have been made for the purpose of exhibiting the range of irregularities to which the comparison is liable. It would have been better in the first instance to choose some place of observation which is not affected by these special complications. There is no place on shore which is free from objection. It was thought that the best available selection would be an inland station like Upavon where at least the differences depending upon orientation are less marked than they are near the coast. Of the stations which were available the one which seemed likely to give the best observations was South Farnborough and in consequence the observations for a year at that station were examined in the Meteorological Office by H. Jeffreys, but it was not found possible to classify the observations in such a way as to obtain a satisfactory clue to the régime of the structure of the layer within the first five hundred metres by the statistical arrangements which suggested themselves. The observations used were those for the early morning about 7 h, a time of day when the curve of variation of wind with height as represented in fig. 1 of chap. IV, p. 29, is in process of changing and perhaps the vertical component may be irregular. The next step in the inquiry seems to be to try a comparison for the simpler conditions of the open sea as soon as suitable observations can be obtained.

The results for the land-stations are tantalising. There is sufficient evidence of relationship to invite endeavours to reach precision and yet, from causes of which proper account cannot be taken at present by those who have before them only the barometric gradient and other surface conditions to guide them, there are many individual cases for which the departures are so large that some means of discriminating between occasions of regularity and irregularity are desirable.

Let us now consider some of the reasons for the discrepancies which are thus observed and from which we may feel that observations in the upper air might be free. We need not revert again to the casual uncertainties of the

observations of wind, and we do not intend to say more about the determination of the gradient except that, for obvious reasons, the computation of the wind from the gradient as shown on a map has little meaning when the gradient is very slight and irregular or when a barograph with an open scale shows notable embroidery.

We must however remind the reader that the comparison is made between individual estimates of the gradient at the surface and the corresponding observations of a pilot-balloon with a single theodolite and these latter are affected by any vertical component that may happen to be in the wind at the time of observation. From Table II of chap. VI it appears that vertical components are specially frequent within the first kilometre of height at Pyrton Hill. Near the sea-coast irregularities arising from this cause are probably still more frequent. It is also probably on this account that J. S. Dines's comparison with the mean value of a group of neighbouring observations gives a higher correlation ratio than the comparison for single stations.

We start from the position that the geostrophic wind can only be expected to be in agreement with the undisturbed wind at its own level and at the time of the map from which the wind is computed. For comparison with the observed wind at 500 metres the geostrophic wind as deduced from the distribution of pressure at the surface requires correction. The correction is by no means negligible. It depends upon the distribution of temperature and, using the formulae which have been developed in the previous chapter, we find that near the surface a horizontal gradient of temperature of 1° F. per 60 nautical miles or 0·5a per 100 kilometres alters the computed geostrophic velocity by one mile per hour within 1000 feet of height. A gradient of temperature of that amount from South to North will add one mile per hour to the component velocity from the West for 1000 feet of elevation and the same temperature-gradient from West to East will make the same addition for the same height to the component of the geostrophic wind *from the North*. In the commoner units of this book the effect of horizontal gradient of temperature upon the geostrophic wind is as follows:

Horizontal gradient of temperature	*Increase* of geostrophic wind-velocity upward
From W. to E. ...	... Component from North
From S. to N. ...	... Component from West
1a per 100 kilometres	... 0·3 m/s per 100 metres

These changes in the components of velocity will appear as large percentages of the geostrophic wind computed for the surface if that wind itself is light. If the gradient of temperature has the same orientation as the gradient of pressure and is large enough to give a steeper slope to the isothermal surface than that of the isobaric surface the geostrophic wind-velocity will *increase* with height without change of direction and, on the other hand, if the orientation of the gradient of temperature is opposite to that of pressure the geostrophic wind will *decrease* with height without any change of direction; but if the orientation of the gradient of temperature is inclined

to the gradient of pressure at any finite angle the direction as well as the velocity of the gradient-wind will change with height.

The extent to which this correction might affect the comparison between the observed wind at 500 metres and the gradient-wind is not easily determined. A fair estimate for stations within the general area of the British Isles may be formed from the maps of the normal distribution of maximum and minimum temperature over the area given in Part II[1]. In January there is an average gradient of maximum temperature from Scilly to Cromer in Norfolk of $\frac{3}{4}$° F. in 60 nautical miles, rather steeper at the South-Western end of the line; for the minimum, or night temperature, the gradient is irregular in the Midland and Eastern counties but over Wales there is a gradient of about 12° F. in 60 nautical miles which would mean a correction of 12 miles an hour to the computed value of the geostrophic wind. In July, on the other hand, the gradients of temperature at night are generally slight and irregular over the inland counties with a fringe of steep gradient from the coast inland amounting to more than 15° F. per 60 nautical miles on the East coast; and in middle day, according to the distribution of maximum temperature, there is a large area of high temperature over the hinterland of England fringed by steep gradients towards the coast-lines which is again most pronounced on the Eastern side and amounts to more than 15° F. per 60 nautical miles on the Norfolk and Kent coasts.

The coastal regions especially those on the Eastern side are thus indicated as peculiarly liable to large corrections to the computed value of the gradient-wind and this applies to the belt ten or twenty miles broad along the coast as represented on a map drawn to a small scale. If we go into detail the gradient would certainly be found to be very pronounced along the extreme fringe of the coast. The steepness of the gradient in some cases is indicated by a diagram in the Meteorological Office representing a section drawn from West to East across the British Isles showing temperature and sunshine during cloudless weather, March 24–30, 1907[2]. It shows an average difference of mean maximum of 13° F., and an extreme difference of 17°, between Geldeston near Beccles, six miles from the coast, and Yarmouth, which would probably work out to give a gradient from Geldeston to the coast at the rate of 130° F. or 170° F. per 60 nautical miles. We are not entitled to suppose that these differences exist throughout the vertical range of 500 metres; if they did the correction to the computed geostrophic wind at a station on or near the coast would be prodigious. But its extension in a modified degree upwards will easily account for a considerable difference between the barometric gradient at the surface and the gradient with which the wind obtained by observations of a pilot-balloon ought to be compared.

These large local differences of temperature are usually called upon to account for sea-breezes which are not amenable to the surface-gradient but it

[1] Specimens of these maps for January and July are given in *The Weather Map*, M. O. Publication, No. 225 i, 1917, p. 88.

[2] *Forecasting Weather*, p. 285.

will be seen that in accepting an explanation in that general form we should be leaving the local details of the gradient out of account.

There is moreover another reason why a station on the coast presents a complication in the relation of observed wind to gradient which may be operative in windy weather when the local gradient of temperature is not very marked. This second reason is the dynamical effect upon the stream of air due to the sudden transition between a surface with a comparatively low coefficient of eddy-viscosity, such as the sea, and one with a comparatively high coefficient, such as a land-surface, particularly a hilly or rugged land-surface. This change must probably be represented by a sudden transition of pressure in the surface layers which produces a "refraction" of the isobaric lines on crossing the coast. We have already referred to it in a note on the diagram on p. 15, and we have mentioned that this view is borne out by the fact that there is a difference of pressure between coastal stations and inland stations, the inland pressure being the higher[1]. Objection has been taken to the view that an increase of pressure over the land can be due to the partial arrest of horizontal motion of the air by the land on the ground that in a "wind channel" there is no increase of pressure upon the surface of a disc exposed to a current in its own plane. It is not clear that the objection holds in the case of wind over land. Even if the multiplication of scale makes no difference to the argument, the relief must cause local differences of pressure and at the coast presumably an increase for on-shore winds; and the mere addition of the volume of the land to that of the air which passes over it must produce some increase of the pressure at sea level.

A form of refraction of isobaric lines has been dealt with in papers which have come from the Meteorological Office on line squalls[2]. In these cases we see on the map a set of nearly straight isobars running in one direction connected abruptly with another set of nearly straight isobars running in another direction. On the map it has been customary to mark the junctions of the lines as simple discontinuities of direction and previously they were rounded off by smooth curves. At the junction there is really a curious dislocation corresponding with the crochet d'orage as represented in the carefully drawn diagrams of the papers referred to, some of which are reproduced in *Forecasting Weather*[3]. In these cases, which correspond with single refraction, the velocity normal to the line of separation of the two fields is maintained though the velocities on either side of the boundary are different. The boundary itself is a locality of great vertical motion of air generally represented by a squall and a shower of rain or hail at the surface.

The refraction which takes place at the coast-line is different in that there is, as we may suppose, no permanent change in the direction of the isobars or

[1] See a paper by Gold, 'Comparison of Ship's Barometer Readings, etc.,' *Q. J. Roy. Met. Soc.*, vol. XXXIV, p. 97, 1908.

[2] R. G. K. Lempfert, 'The Line Squall of Feb. 8, 1906,' *Q. J. Roy. Met. Soc.*, vol. XXXII, p. 259. R. G. K. Lempfert and R. Corless, 'Line Squalls and Associated Phenomena,' *ibid.* vol. XXXVI, p. 135.

[3] *Loc. cit.* pp. 239–243.

their separation. We may suppose the effect to be a dislocation of the isobars giving a figure (fig. 1) somewhat similar to that representing the refraction of light by a plate. What happens in the area represented by the belt of coast is at present undetermined, but certainly some complications of the flow of air result: the cliff-eddy is in certain cases one of them and in hilly country other eddies are recognised.

This is doubtless a highly speculative method of treating the difficult question of the effect of the coast-line on the geostrophic wind but there are certain facts which seem to be in favour of it.

There is the high degree of divergence of the wind from the isobar at Southport noticed in chap. II which would be accounted for by a deviation of the isobars themselves through a suitable angle. Suitable angles for different orientations are represented in fig. 1, here. If that is the true explanation of the phenomenon the effect would seem to be most marked at Southport when the wind is along the coast and the land is on the right. That peculiarity may be due in that instance to the special shape of the Lancashire coast. Readers will be familiar with the manner in which the lines of the coast on either side of the English Channel are marked by cumulus clouds on days when a Westerly wind blows along the Channel, bringing with it probably air which is tending towards instability before it has to meet with the mechanical interference of the coast-lines.

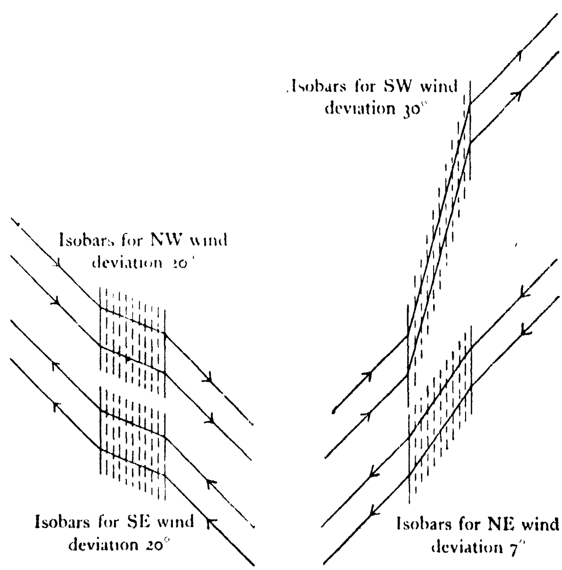


FIG. 1. Refraction of isobars at Southport computed from the data represented in figure 2, chapter II, p. 15.

The transition from land to sea must have its counterpart in the transition from sea to land, but whereas it is easy to think of thermal effects such as the formation of clouds by the piling up of air in consequence of the arrest of its motion on coming from sea to land, no such easy expression of the transition from land to sea occurs to us. Information has recently come from the North East coast of a "barrage" which makes the manœuvring of airships difficult or even impossible in a Westerly wind leaving the land, but the information is not in sufficient detail to indicate how far the barrage may be accounted for by the mechanical effect of the refraction of the isobars crossing the coast nor what part of the effect is due to the enhancement of the eddy-motion by hills in the immediate neighbourhood and the consequent increase

in the effective resistance opposed by the air to the travel of an airship. So far as we know no experiments have been made to compare the forces exerted upon an obstacle by an eddy stream with those of a stream of the same mean velocity without eddies.

We are not yet in a position to apply numerical corrections to the computed geostrophic wind in individual cases on account of the local temperature-gradient or the refraction of the isobars by the coast-line, but both causes must combine in any actual case, and there seems no reason to consider that the discrepancies actually noted between the observed wind at 500 metres and the geostrophic wind at the surface are due to any real finite difference between the wind in the free air and the geostrophic wind for the same level.

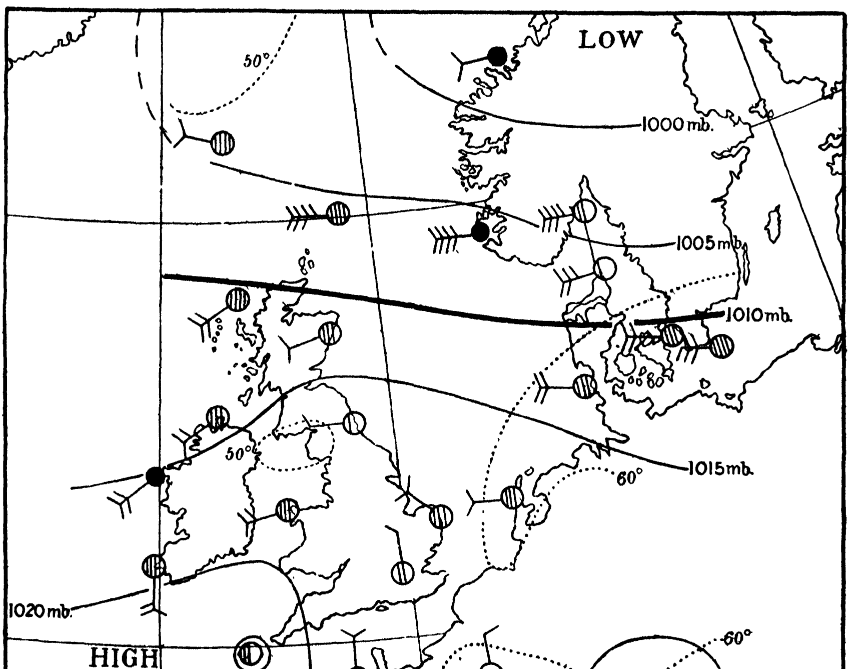


FIG. 2. Chart of Isobars, Winds, and Isotherms for 7 h from the Daily Weather Report for August 16, 1918.

The full lines are isobars, the dotted lines isotherms.

The arrows fly with the wind. The number of feathers indicates the force on the Beaufort Scale. The small circles represent the positions of the stations: the number of cross bars within the circles indicates the number of quarters of the sky covered by cloud. A black dot shows that rain is falling.

Even when all allowances are made there remains the difficulty of drawing isobars with sufficient precision. We give an example of the map for 7 h on August 16, 1918, in which the adjustment of isobars to winds is almost inexplicable as it stands and may well be offered as an exercise for the student.

CHAPTER IX

Synchronous charts of horizontal motion in the free air

THE multiplication of observations with pilot-balloons, which give the direction and velocity of the air in a horizontal plane at successive levels, has provided a large amount of new material in aid of the study of the structure of the atmosphere, and of the variations of structure in the vertical columns into which the whole atmosphere in the region under investigation may be divided for the purpose of "diagnosis," adopting a word used by V. Bjerknes who may be regarded as the principal authority on this subject. Hitherto we have dealt exclusively with the vertical column at a particular station and we must now extend our inquiry by bringing together our knowledge of the vertical columns corresponding with a series of stations in various parts of the country.

The first step in the diagnosis is naturally to plot all the observations for the same level using a separate map for each separate level, just as we are accustomed to refer to a map of the distribution of pressure at sea level the meteorological observations at all the available stations at the surface. The plotting has been done now for some months at the Meteorological Office where the winds for five different levels, as obtained from observations of pilot-balloons at upwards of thirty stations, are set out on maps. The direction of the wind at each level is indicated on the chart for that level by an arrow drawn to fly with the wind, as usual, and the velocity by figures inserted in small circles circumscribing the points which mark the positions of the verticals at the respective stations.

The next step is to compare the figures for each station with those for the surrounding stations. The same process is regularly followed in preparing the customary map of the distribution of pressure at sea level and the distribution of temperature at the surface. By that means the observations of pressure are subjected to a rigorous scrutiny and any outstanding exceptional reading is the subject of immediate inquiry by telegraph. The observations of temperature are similarly scrutinised and occasional errors of five degrees or ten degrees are sometimes corrected in that way. The observations of the state of the sky are practically passed without scrutiny because local cloud in a region of generally clear sky, or the reverse, is not regarded as an improbable phenomenon. And in like manner the observations of wind are not questioned unless the divergence of the reported wind from the direction or force which the gradient would lead us to expect is very marked and the situation of the station is known to be consistent with an approximate relation to the gradient. Very large deviations from the apparent gradient either in direction or force are accepted as real for stations in Iceland, the Faroë and Norway or for high

level stations on the continent, and considerable deviations are allowed to pass without challenge for some of our own stations. Some stations indeed are recognised as having a peculiar local bias for which an appropriate explanation is being gradually sought.

When we come to deal with the comparison of the winds in the upper levels as plotted from observations with pilot-balloons we find sometimes considerable deviations, occasionally in direction but more frequently in the figures representing the velocity. Here a difficulty presents itself arising from the fact that the observations are made with only one theodolite. We have seen that if the balloon happens to be in an ascending air-current the horizontal velocity as computed from the observations will be too small by the amount represented by $w \cot E$, where w is the vertical component of the velocity of the air and E the angular elevation of the balloon. We must also reckon with occasional errors of reading which have the same effect upon the computed velocity as a vertical component of the motion of the air. In the circumstances there is *prima facie* no indication of the real explanation of an exceptional reading.

In dealing with observations of pressure the isobars furnish, as a rule, a very satisfactory means of distinguishing between errors of reading and local meteorological peculiarities because the continuity of the distribution of pressure must be expressed in the isobars, but there is no such necessary continuity in the distribution of wind-velocity, although, for all ordinary circumstances, we postulate a relation between the wind and the distribution of pressure.

For the maps representing observations with pilot-balloons we have no observations of pressure in the upper air with which the wind can be compared; we have to deal with the observations of wind alone.

Very little has been done in practice in the study of maps of wind observations at the surface as a separate part of meteorology. Professor V. Bjerknes[1], formerly of Christiania, has opened up a method of dealing with those observations in two volumes on Dynamic Meteorology and Hydrography, in which are described and illustrated novel methods of dealing systematically with observations of meteorological elements grouped in maps, devised and developed with the aid of several collaborators. Bjerknes's plan is very comprehensive. After defining his variables, of which those regarded as independent are the coordinates defining geographical position and height together with the time, and, omitting provisionally the influence of electric or magnetic fields, those required for the description of atmospheric states are five meteorological elements, namely, pressure, mass, temperature, humidity and motion, with a corresponding set of five for hydrography, he states concisely the problem of meteorology and hydrography: *To investigate the five meteorological elements and the five hydrographic elements as functions of the coordinates*

[1] *Dynamic Meteorology and Hydrography*. Carnegie Institution of Washington. Publication No. 88. Part I, Statics, by V. Bjerknes and J. W. Sandström, 1910. Part II, Kinematics, by V. Bjerknes, Th. Hesselberg and O. Devik with separate volume of Plates, 1911.

and the time. He distinguishes between the Climatological Method which consists in giving constant values to the coordinates and examining the effect of letting time vary (using, for example, the results of automatic recorders of the various elements set up at fixed points) and thus obtaining the normals and the periodic or secular or irregular variations, and the Dynamic Method which, using the same records, depends upon a series of synchronous representations of the field of each meteorological element. He thus presents for examination a series of pictures of the state of the atmosphere for a succession of chosen epochs. "The comparative investigation of the successive states must lead to the solution of the ultimate problem of meteorological or hydrographic science, viz., that of discovering the laws according to which an atmospheric or hydrographic state develops out of the preceding one." It is this method which is treated in the volumes referred to.

The method is called dynamic because, "in virtue of the laws of hydrodynamics and thermodynamics which govern atmospheric and hydrospheric phenomena, preceding states are in relation of causality to subsequent states. Inasmuch as we know the laws of hydrodynamics and thermodynamics, we know the intrinsic laws according to which the subsequent states develop out of the preceding ones. We are therefore entitled to consider the ultimate problem of meteorological and hydrographical science, that of precalculation of future states, as one of which we already possess the *implicit* solution, and we have full reason to believe that we shall succeed in making this solution an explicit one according as we succeed in finding the methods of making full practical use of the laws of hydrodynamics and thermodynamics[1]."

These quotations illustrate the clearness with which Bjerknes treats the questions which he considers, a clearness which is equally conspicuous in his general examination of the results of the available methods of observation. The two parts already published present the treatment from the statical and kinematical standpoints respectively. They are accompanied by a series of tables for pursuing the necessary calculation. The dynamical treatment is to follow, but graphical methods for performing the mathematical operations are given in chaps. VIII and IX of Part II, Kinematics. "These will be of the same importance for the progress of dynamic meteorology and hydrography as the methods of graphical statics and graphical dynamics have been for the progress of technical sciences."

We have given an outline of Bjerknes's scheme for dealing with the general meteorological problem because it represents a systematic attempt to organise and combine meteorological observations in such a way as to lead directly to the inference by mathematical operations of the sequence of states of the atmosphere. The method which he puts forward is graphical; the reasoning is to be applied to a series of maps or diagrams of the distribution of elements in a set of horizontal surfaces separated by equal differences of "dynamic" level, that is to say the successive surfaces of equal geopotential, not necessarily of equal geometrical height, or some other series of surfaces defined by

[1] *Loc. cit.* §§ 87 to 90 (Part II, chap. I).

selected values of one of the variables. In that way a complete "diagnosis" of a succession of states of the atmosphere will be obtained which can be related the one to the other by mathematical process.

The same idea, that if we are sufficiently acquainted with the facts and competent to deal with them on the basis of Newtonian dynamics the causal relations of the sequence of states must be disclosed, has been taken up by L. F. Richardson and pursued to the point of dynamical operation in a work on *Weather Prediction by Numerical Process*, which is now in course of publication by the Cambridge University Press. The matter interests us here because at the bottom of all the possibility of calculation lies the assumption that the data which form the basis of the maps or the arithmetical process are a complete representation of all the pertinent facts of the state of the atmosphere for the purpose of mathematical treatment. Bjerknes, for example, suggests that an effective organisation in regard to time intervals for pressure and temperature in the upper air would be continuous observation or observations every hour of Greenwich time at all stations at the ground, ascents at every third hour of Greenwich time from pilot-balloon stations and for every sixth hour of Greenwich time from the complete aerological stations. We may note in passing that to make the "diagnosis" complete these observations would need supplementing by corresponding observations over the sea which would require special organisation.

To anyone who has spent many years of his life in the bewildering occupation of compiling and arranging the multitude of figures and symbols which are collected for the purpose of representing the state of the atmosphere over land and sea for the hour, the day, the week, the month, the year, or a series of years, the knowledge that more than one student of the atmosphere feels that the figures are, or can be made to become, capable of arrangement in such a way as to invite a general attack upon the whole problem is very encouraging. The alternative which presents itself to those who are apprehensive that mathematical operations only develop the ideas which are intrinsically implied by the process of selecting the data is to use a different method of selecting the data, to watch for occasions when the natural phenomena arrange themselves in a manner which points to a definite classification or grouping. Reasoning may then be applied to special groups of facts that have real existence for a sufficiently long period to furnish a definite mental picture even if the grouping be not apparent on other occasions and therefore not strictly speaking general. In other words, we select examples for which a special train of reasoning may be improvised rather than prescribe beforehand the course of reasoning to be applied on all occasions with the condition that the data shall be so organised and selected as to make the prescribed course of reasoning applicable.

For no element is the difference of standpoint more easily illustrated than the motion of the free atmosphere at different levels. The measure of the motion is a vector quantity, its direction and speed must be defined. Bjerknes represents a field of atmospheric motion by means of instantaneous lines of

flow, or "isogonal" lines instead, and lines of equal speed. In this way the field of motion of the air on any occasion is completely mapped. The mapping of a large area for air-motion will generally disclose a series of lines or points of convergence or divergence of the instantaneous lines of flow with which must be associated instantaneous upward or downward motion.

Taken in successive layers the whole of space would be filled with a solenoid of tubes of flow of which the lines in the successive planes represent the sections and, according to the work of Sandström, a particular pattern of lines of flow in any plane can be associated with atmospheric motion in three dimensions of recognised character such as wave-motion, or the convergence of a cyclone or the divergence of an anticyclone.

For those who are unfamiliar with this mode of procedure the patterns formed by the lines of flow for selected types of motion are sometimes sur-

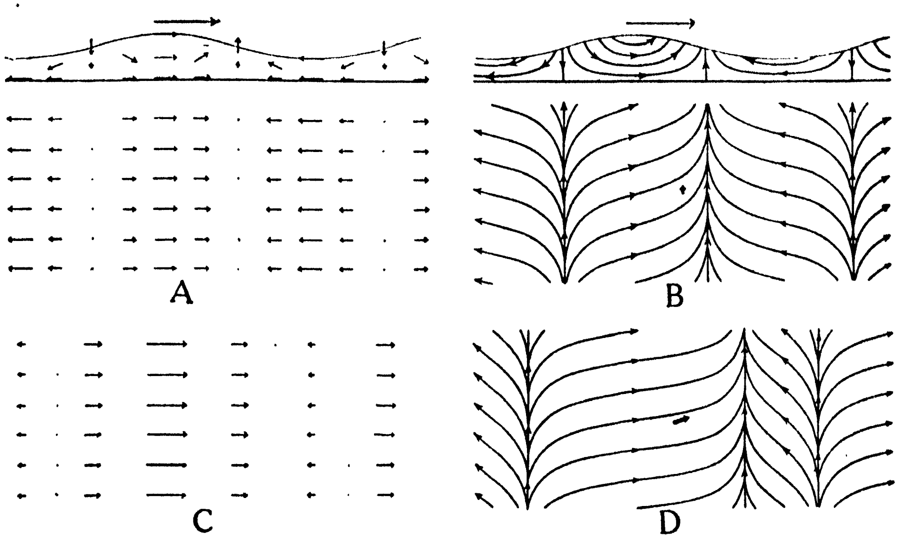


FIG. 1. Wave-motion crossing a current of air. (From Bjerknes *Dynamic Meteorology and Hydrography*, Part II, chap. v.)

prising and they are useful to us because they remind us that the information contained in a chart of instantaneous motion, of which a synchronous weather-map is an example, may not at first sight disclose all the information implicitly contained in it. We have taken the liberty of reproducing in fig. 1 four diagrams A, B, C, D, representing wave-motion travelling across a current of air. As represented in the first pair of diagrams A, B, the train of waves is travelling in the direction of the arrow at the top directly across the flow of the air-current. Diagram A comprises two figures representing the wave-motion in horizontal and vertical section and diagram B the lines of flow in horizontal and vertical section for the wave-motion combined with the motion of translation of the current which is represented by the small thick arrow near the middle of the diagram. Of the other pair of diagrams D represents

the horizontal lines of flow for the same wave-motion combined with a translation, also represented by an arrow, oblique to the direction of advance of the wave, while C gives the components of motion in the direction of advance of the wave. The transverse lines in the resultant diagrams show the lines of convergence and divergence where there is no horizontal motion in the waves. There is however vertical motion in the wave and the horizontal motion of translation. The crests and hollows of the waves are regions of maximum horizontal velocity in opposite directions.

These diagrams are reproduced because the study of wave-motion in the atmosphere is one of great interest and many meteorologists are unaware that it would be indicated by a set of observations with pilot-balloons which, when plotted on a map, conformed to one of the varieties of the patterns represented; and they make clear that the systematic study of atmospheric data as proposed by Bjerknes is a practical part of meteorological science. The peculiarity of wave-motion is that, provided conditions for the transmission of waves exist, it may be found very far away from the locality where it was produced and may therefore be observed quite independently of any local exciting cause. It has already been pointed out in chap. V with reference to the photographs of clouds attributed to eddy-motion that the forms of the lenticular clouds suggest a combination of wave-motion and translation such as that represented in fig. 1. Many other typical forms of motion are represented for which reference should be made to the original work.

If the motion were confined to two dimensions so that each plane could be regarded as independent of the other planes we could map the field of each level as a solenoidal field in which the separation of the lines of flow is inversely proportional to the flow measured in terms of momentum. If we map the field on this hypothesis, we get of course into difficulties wherever the air-currents have a vertical component. Using a familiar analogy those readers who are acquainted with the practice of a physical laboratory will recollect that the same kind of difficulty arises in the use of lines of force to represent the intensity as well as the direction of the horizontal magnetic force due to a bar magnet in the earth's field.

The other elements are scalar and a single set of lines is sufficient to represent the field. Thus barometric pressure is scalar but the barometric gradient is a vector and in meteorological practice we are accustomed to associate closely the vector wind with the vector gradient, so that having a map of the distribution of pressure we should not hesitate to draw a map of the winds which from experience we should expect to be quite as effective in representing the winds, or even more effective than one which was based upon a series of actual observations of winds at a limited number of stations say 100 kilometres apart. Our map of the winds so drawn according to the isobars would certainly be in difficulties at any point where there was vertical motion, but whether it be that the vertical component of motion is generally so small that its effect does not seriously interfere with the local wind or that it is so local

or transient that its effect is not noticed in our maps, experience has not taught us to distrust the gradient as an indication of the wind. By assuming the relation between the horizontal wind and the horizontal distribution of pressure in the free air as a first law of atmospheric motion, we are really assuming that those portions of the field where vertical motion or other complications invalidate the relation are to be left out of the survey or postponed for future consideration while the parts of the field where the law is practically applicable are being dealt with. But if the distribution of pressure enables us to draw a map of the winds then equally a map of pilot-balloon observations, in so far as they give us a correct representation of the horizontal winds, ought to enable us to form a working idea of the distribution of pressure. If we could fill the map with a picture in two dimensions of a solenoidal distribution of wind-velocity it ought to afford a good representation of the distribution of pressure. We may therefore attempt to deal with the observations of winds at different levels by fitting to them a system of instantaneous lines of flow for which the flow measured in terms of momentum is everywhere inversely proportional to the separation of the lines. We make in fact a map of the "stream-function" instead of a map of pressure. The difficulty in drawing a field of lines of flow of this kind from the charted observations of pilot-balloons is that the wind gives us only the space rate of change of pressure, and for each station we can only calculate the separation of consecutive isobars, not the actual position of either. The completed chart would in effect give us information as to the velocity at every point of the map of many of which we have no knowledge. We cannot approach the solution step by step; we can only by trial submit a complete solution that fits the facts at the points of observation.

In making the attempt we are not discouraged by the fact that the actual treatment is not as rigorous as that proposed by Bjerknes or Richardson. The course which we propose depends upon assumed relations which are not verified. Either system ultimately depends on an ideal. Bjerknes sets aside the ideal of motion which we have taken as the first law of atmospheric motion with the remarks "The accordance of these curves (the isobars) with the direction of the arrows (representing the stream lines) is never complete and should be complete only in exceptional cases." "Further, the numbers representing the observed wind-intensities are never in full accordance with the formula." But he uses the acknowledged relation between wind and gradient as an auxiliary to draw lines of flow by making them cut the isobars at certain angles. And he draws the curves of equal wind-intensity so as to get certain departures from the theoretical value[1]. In view of what has been adduced in chapters II, III and VIII, this process seems to be based upon an unsatisfying ideal. It is not unfair to say that if we are to assign any definite value to the deviation of the wind from the isobar or any definite value to the ratio of the wind to the gradient-wind (for straight isobars) in the free air, when we have not actually measured them, the only value of the deviation and the only value

[1] *Loc. cit.*, Part II, p. 62, § 139, 'Dynamic diagnosis of motion in the Free Space.'

of the ratio that have any substantial claim are zero and unity respectively. And, moreover, in giving practical directions for constructing a picture of the field of motion in the atmospheric space near the ground, Bjerknes suggests[1] that "a point of divergence will appear where there is a descending current (centre of anticyclone) and a point of convergence where there is an ascending current (centre of cyclone)," and among the supplementary rules for obtaining the lines of flow from a limited number of observations of wind he writes "Within a barometric depression there is a probability for existence of points or lines of convergence; within areas of high pressure there is a probability for the existence of points or lines of divergence. Long ridges of high pressure will as a rule contain a line of divergence; long ridges of low pressure a line of convergence." This ideal as the basis of a representation of the atmosphere is unsatisfying for three reasons:

First, after most careful inquiry, as set out in the *Life-History of Surface Air-currents*, the central regions of anticyclones did not manifest themselves as regions of descending air but as masses of the atmosphere of great stability which apparently took no part in the supply of air to the surface. Secondly, as set out in our discussion in Part II, anticyclones represent indeed regions of relative concentration of entropy which must somehow be disposed of if the air is to get downward. If an anticyclone were really a large column of air descending from some upper level to the surface the air within it would be in labile equilibrium just as a column of ascending air is, and would therefore be liable to show the effects of instability rather than stability; and thirdly, the ascent of air in the central region of a cyclone, as set out in the *Life-History*, is not a necessary accompaniment of the existence of the cyclone and the apparent convergence to the centre is very much modified by the motion of the centre itself. An instructive commentary on the conventional view that a cyclone is a region of upward convection whereas an anticyclone is a region of downward convection may be found in the fact that the tropopause, the lower limit of the stratosphere, is higher over an anticyclone than over a cyclone. Convection therefore reaches a higher limit and the cooling which is a natural consequence of upward convection is carried further up in an anticyclone than in a cyclone.

In formulating his method Bjerknes uses great precision as to the processes to which the pictures of the atmospheric fields are to be subjected, but in forming the pictures he accepts without inquiry the traditional ideals of meteorological situations, which doubtless have some foundation in fact but when examined in detail are not in accordance with all the known facts, and have in experience proved to be an insuperable hindrance to the progress of meteorology in the past fifty years.

We may acknowledge with thankfulness Bjerknes's directions for the method of using observations of wind in the upper air to form a map of the field of motion without accepting in detail the aid which he proposes to derive from ideals which may be applicable in a generalised sense to the atmospheric

[1] *Loc. cit.* p. 48.

processes on the large scale, but which seem to us less appropriate than our own to the closer study necessary for the practical application of the results of soundings with pilot-balloons. The difference in practice between the two methods is that when two sets of lines are used to map the field the lines of flow can be drawn without regard to number, but to represent the field by one set of lines implies drawing them at calculated distances.

After this digression in justification of a method which fortunately seems most free from objection for those parts of a map where pilot-ballooning is practicable, that is to say, where there is comparative freedom from low clouds, vertical motion, and other characteristics of the neighbourhood of many of the singular points or lines of a field of air-motion, let us confess that the discussion of methods is by far the easiest part of the whole programme. The practical application is beset with minor troubles and is very laborious considering that maps for many levels three times a day at least come up for consideration. We have first to form a general idea of the way in which the lines run and then decide whether exceptional values of direction and velocity shall be regarded as mistakes of reading and ignored or as localities of ascending or descending currents which, having regard to the scale of the map, may also be ignored in drawing the lines and indicated with some special symbol on the map, or thirdly, as marking some change in atmospheric structure which ought to be adequately represented in the finished map. Next a number of trials have to be made as the only guides are the direction and separation of the lines at certain points. Drawing lines in pencil, and rubbing them out again as may be required, is the plan adopted in drawing isobars, and is workable enough because each line can be separately and finally decided upon. But with lines to represent tubes of flow a whole line may have to be displaced with some slight modification of shape in order to accommodate the other lines in its neighbourhood. The only workable plan which has so far suggested itself is to use small squares of card of standard size which can be moved about the map and laid side by side to the proper separations and flexible wire for trial lines which can be bent to the proper shape, laid down on the map and moved bodily when necessary. When the field has been approximately mapped in this way final lines can be drawn in.

The observations of velocity in successive levels of 1000 feet come to the Office expressed in miles per hour with the direction given in points. The setting out of the lines therefore requires a table of distances of separation for winds of specified velocity for given values of the density which, in the absence of direct observations of pressure and temperature, must be taken from a table of normal densities at the different levels in the several months. On the average for the whole year these values expressed in grammes per cubic metre are approximately 1250 at sea level, 1050 at 5000 ft., 900 at 10,000 ft. and 775 at 15,000 ft. and 650 at 20,000 ft. The table of separation for intervals of five miles per hour which will correspond with the separation of isobars for steps of 5 mb. in latitude 52° and for these densities will be as follows:

TABLE OF SEPARATION IN NAUTICAL MILES OF THE LINES OF FLOW IN SOLENOIDAL WIND-CHARTS FOR DIFFERENT VALUES OF VELOCITY IN MILES PER HOUR AND DENSITY IN GRAMMES PER CUBIC METRE.

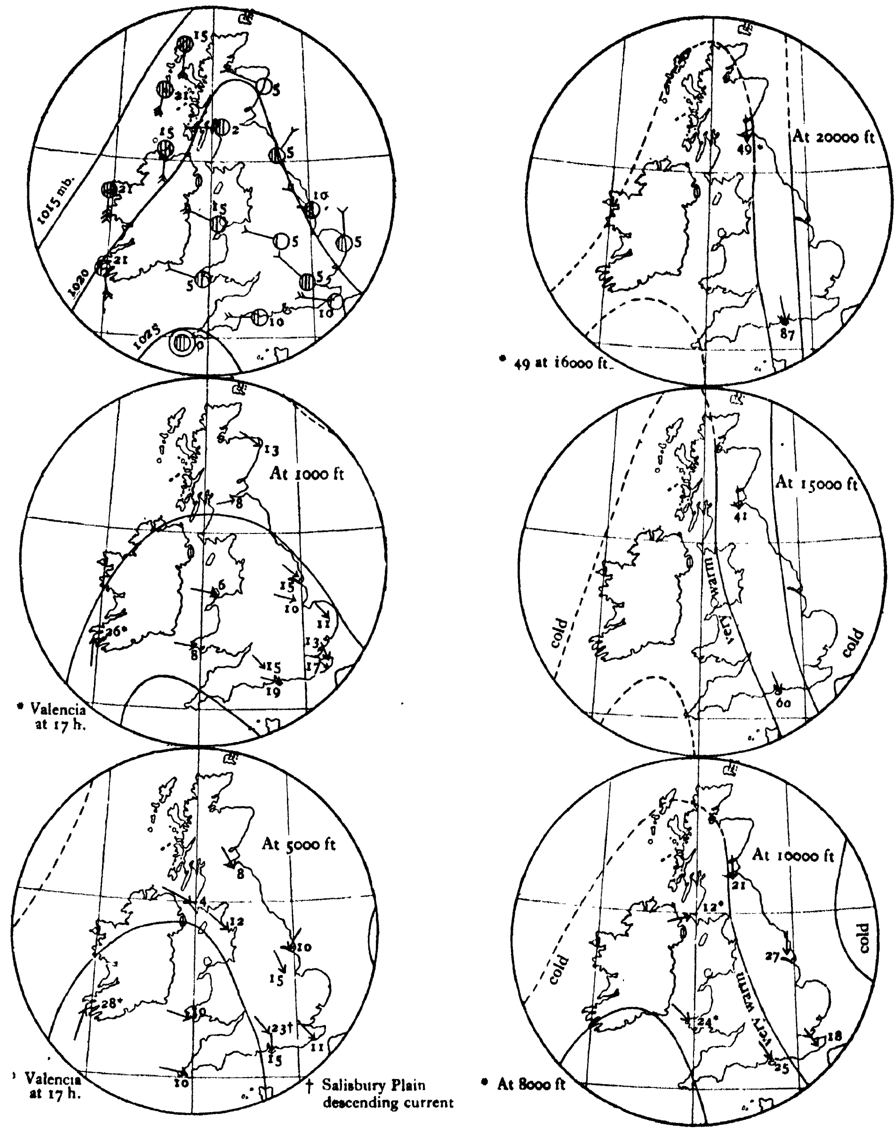
Velocity		Selected values of density in grammes per cubic metre				
		1250	1050	900	775	650
m/s	m.p.h.	Distance of separation of lines of flow in nautical miles				
2·2	5	845	1006	1174	1362	1625
4·5	10	423	503	587	681	812
6·7	15	282	335	391	454	542
8·9	20	211	251	293	341	406
11·2	25	169	201	235	272	325
13·4	30	141	168	196	227	271
15·7	35	121	144	168	194	232
17·9	40	106	126	147	170	203
20·1	45	94	112	130	151	181
22·4	50	85	101	117	136	163
24·6	55	77	91	107	124	148
26·8	60	70	84	98	114	135
29·1	65	65	77	90	105	125
31·3	70	60	72	84	97	116
33·5	75	56	67	78	91	108
35·8	80	53	63	73	85	102
38·0	85	50	59	69	80	96
40·2	90	47	56	65	76	90
42·5	95	44	53	62	72	86
44·7	100	42	50	59	68	81

We may give in illustration of the application of this method maps representing the conditions at the surface and the distribution of velocity at 1000, 5000, 10,000, 15,000 and 20,000 feet on October 19, 1917, referred to in chap. VI when a very strong wind in the upper air carried a fleet of Zeppelins out of their course. In this set of maps the distribution of velocity is represented, not the distribution of momentum; the pressure-difference corresponding with the separation of the lines is 5 mb. at the surface; in the upper levels it is less in the inverse ratio of the densities.

The maps of the flow of air at the different levels at about midday or 13 h of October 19, 1917, represented in fig. 2 are appropriate to the meteorological conditions of a depression with strong southerly winds advancing in the rear of another depression which is passing eastward. They make clear the special feature of the occasion, viz., a very large increase in the northerly wind in the extreme rear of the passing depression between 5000 feet and 20,000 feet, more especially between 10,000 feet and 20,000 feet. Observations near Portsmouth extended to those heights and those near Edinburgh up to 16,000 feet. They are as follows:

Height in feet ...	5000	10,000	15,000	16,000	20,000
Wind-velocity near Edinburgh in miles per hour	NW 8	N 21	N 41	N 49	—
Wind-velocity near Portsmouth in miles per hour	—	NNW 25	N by W 60	N by W 65	N by W 87

FIG. 2. MAPS OF THE DISTRIBUTION OF AIR-FLOW AT 20,000 ft., 15,000 ft., 10,000 ft., 5000 ft., and 1000 ft., together with a map of the isobars at sea level and winds at the surface at or about 13 h on October 19, 1917.



Note. The general scheme of the lines is based on the meteorological situation indicated by the weather-maps of the day which shows that a depression was advancing from the West. Later in the day at 17 h strong southerly winds were indicated at 1000 ft. and 5000 ft. at Valencia. The direction of the flow is shown by arrows and the velocity in miles per hour is marked in figures. The lines of flow are drawn so that the velocity of flow is inversely proportional to the separation of the lines, a separation of 60 nautical miles indicating a velocity of 70 miles per hour. Dotted lines indicate a general idea of the flow outside the region of direct observation.

Observations which are not in good accord with the lines are marked; so are those which are not strictly applicable for the drawing of the lines on account of some difference

The maps show that the strong northerly current lay over a North and South band down the middle of England at the time of the maps when the winds near the surface were the light westerly winds of the wedge of high pressure between the two "lows." At 10 o'clock of the following morning a strong northerly current of less velocity was shown in the layer from 14,000 to 20,000 feet over Northern France with very light south-easterly or south-westerly winds beneath. The velocity at 20,000 feet in this strong northerly current was 47 miles per hour. Judging by the rapidity with which the lost airships travelled, the velocity in that region during the previous night or early morning was stronger than that recorded at 10 o'clock; the stronger current had probably by that time passed eastward. The rate of drift eastward may have been that shown by the westerly wind in the wedge between the departing and advancing depressions.

According to the theory which has been set out in chap. VII the rapid increase of velocity with height must be attributed to a steep horizontal gradient of temperature from West to East in the layers between 10,000 feet and 20,000 feet. According to the formula ($5a$) of chap. VII the gradient of temperature required for an increase of velocity of 5 miles per hour per thousand feet at the level of 10,000 feet would be about 1·8 a or 3·3° F. per hundred kilometres. Such a gradient could only arise if the temperature of the wall of air forming the western boundary of the current were nearly isothermal for a thickness of a kilometre or more because the reduction of temperature on the eastern side is limited by the lapse of temperature with height. The lapse cannot pass the adiabatic limit and is normally not far from it so that there is not much margin with which to produce an exceptionally large horizontal gradient. In other words the conditions require a wedge or tongue of air which approaches the isothermal condition or perhaps goes beyond it forming an inversion. Air of this character in the upper regions may be called "very warm"; it would lie between the lower temperature of the air of the passing low and that of the approaching low. The intermediate regions have accordingly been marked "very warm" on the maps for 10,000 feet and 15,000 feet, while "cold" has been written over the regions of the lower pressures on either side of the warm tongue in order to give an idea of the distribution of temperature necessary for the observed phenomena. The belt of very warm air with the rapid current on its right must have travelled eastward. So far as we can tell from the map for the level of 5000 feet the air underneath the warm tongue and also that to the West of it was travelling with a velocity of about 10 miles per hour from the West. At 10,000 feet in the corresponding region there is a velocity of 12 miles per hour from a point South of West; and if we can assume that the whole system, consisting of the strong current with a low on either side, was moving from West to East with a velocity of about 10 miles per hour the tongue of warm air would have travelled about 200 miles to the East between 13 h on the 19th and 10 h on the 20th. If this view is correct the "relative motion" of the strong northerly current would be the actual motion as mapped,

modified by the vector subtraction of a west component of 10 miles per hour.

It so happens that we have a record of temperature in the upper air, for 10 h of the 20th, from Ipswich, about 200 miles to the east of the locality assigned as very warm at 13 h of the previous day. With all the assumptions which we have made we therefore expect to find a nearly isothermal column over Ipswich. The temperatures of this ascent are therefore of peculiar interest and we find as follows:

Height in feet	6350	8350	10,350	12,350	14,350	16,350
Temperature F.	32	30	25	20	16	16

It is certainly remarkable that the air-column from fourteen to sixteen thousand feet is actually isothermal. Below those levels lapse-rates are normal except for the first step quoted.

Once more, this may be a coincidence but it is a very surprising one, and all the more surprising because the strong northerly current measured in Northern France at the same hour only began at 14,000 feet instead of 10,000 feet as it did at Portsmouth on the previous day.

If these facts are more than a coincidence and are indeed what they appear to be, a justification of the assumptions which we have made, it would follow that convection must have invaded the layers over Northern France and Ipswich between 10,000 feet and 14,000 feet within the 21 hours between the time of the map and the time of the observation of temperature, and have demolished the strong northerly current at the same time that it destroyed the gradient of temperature. And that is not surprising, because the formation of an isothermal layer several thousand feet thick must require very exceptional conditions. They exist, we know[1], when the air is left to the influence of radiation and free from convection, and we have no other explanation for an isothermal layer. It would appear, therefore, that this tongue of "very warm" air must have been the survival of a mass of air travelling slowly over the Atlantic from West to East and free from convection long enough for isothermal conditions to be set up and gradually worn away from below to the extent of 4000 feet of its thickness while it passed across England. It had a cyclonic depression on its East side and another on its West side, but whether they invaded it laterally we cannot say.

It is clear that a knowledge of the distribution of temperature at successive levels would be of great assistance to us in the task of preparing synchronous charts of horizontal motion in the free air because they would help to guide our judgment in combining the direct observations of the wind. Observations of temperature at levels up to 20,000 feet are now possible with aeroplanes and it is time that an endeavour be made to incorporate observations of temperature with observations of wind-velocity at those stations which aim at providing means of guidance in aerial navigation. A preliminary difficulty

[1] E. Gold, 'The Isothermal Layer of the Atmosphere and Atmospheric Radiation,' *Proc. Roy. Soc.* A, vol. LXXXII, p. 43, 1909.

arises from the fact that the heights at which the observations are made are not given with the accuracy that is desirable by the ordinary means of observation of height in an aeroplane, but even if that difficulty should prove insuperable in view of the slight inclination of the isobaric surfaces in ordinary conditions of weather we may still be able to identify the localities of exceptional horizontal gradients of temperature which would form the most important features of the maps. Apart from the fact that the slope of the isothermal surfaces is generally steeper than that of the isobaric surfaces we know that there are regions where the air is marked by isothermal conditions or by inversions of lapse with height and corresponding conditions are not possible with pressure. Consequently a continuous record of temperature in relation to pressure would identify these exceptional regions and help materially in setting out a map. In this connexion we may refer to the diagrams of temperature at different heights in the atmosphere on consecutive days obtained by observations with kites. They began with Teisserenc de Bort[1] who prepared one for Paris from observations made at his observatory at Trappes and were continued at Lindenberg[2] for which a year's observations were published in separate form and subsequently at Mount Weather near Washington[3]. They all show very striking changes of temperature setting in and lasting for some days which could easily have been identified by observations in aeroplanes made in appropriate localities.

[1] 'Sur les caractères de la température dans l'atmosphère libre au dessus de 10 kilomètres.' *Proc. verb. de la Commission pour Aérostation Scientifique*, St Petersburg, 1904, p. 110.

[2] Dr R. Assmann, *The temperature of the air above Berlin from October 1st, 1902, until December 31st*, 1903. Berlin, 1904. Otto Salle.

[3] William R. Blair. Free Air Data at Mount Weather. *Bulletin of the Mount Weather Observatory*, vol. IV, pp. 176 et seq., 1912.

CHAPTER X

Curved Isobars

HITHERTO no account has been taken of the second or cyclostrophic term in the equation (γ) of p. 1, which gives the relation between wind and pressure for the motion of the air under balanced forces. It represents that part of the gradient which is balanced by the deviation of the path of the air from a great circle; its numerical importance is inversely proportional to the radius of the small circle which osculates the path of the air and is therefore directly proportional to the curvature of the path in the horizontal plane. In ignoring the effect of curvature we have supposed that the numerical value of the term is, in general, sufficiently small for that course to be followed in view of the uncertainties of the measurements of the wind in the upper air. It has been convenient for us to ignore it because the determination of the curvature of the path is not possible when we have only a map for a single epoch, and what we have written hitherto has dealt with the features of the single map. The fact that the velocity of the air at the particular epoch can be fitted into a scheme of velocities arranged as lines of flow in the form of circles or spirals is not evidence that the circle or the spiral itself represents the path of the air. It can only do so if the features of the map remain stationary. We cannot assume that they do so without referring either to previous maps or subsequent ones, and whenever we make the reference we find that the condition is not exactly fulfilled, generally speaking not even approximately so. There is nearly always a considerable change in the distribution of pressure and wind except in those regions which are represented by the isobars of a large anticyclone.

We now pass on to consider the effect of the curvature of the path upon the relation of wind to the distribution of pressure and our first step shall be to consider what the curvature of the path means for us. The maps which we use to represent the state of the air at any epoch are apt to mislead us unless we are careful. The lines of flow, which we have considered in the previous chapters to be in agreement with the isobars in the upper air and to cross the isobars at the surface at certain finite angles depending upon various conditions of turbulence, are represented by curves with a very great variety of curvature, but the lines representing the paths of air whenever they have been constructed are not at all likely to be mistaken either for lines of flow or for isobars unless we happen to be dealing with a part of the map where the isobars are straight and parallel, and the lines of flow either lie along the isobars or cross them at a uniform angle. On the other hand there is nothing in the appearance of a line of flow or an isobar that disqualifies it as a path. It becomes, in fact, a path if the situation is permanent, but that means a set

of conditions for which, so far as we know, there is no rigorous example in our maps. A near approach is represented in the series of maps for the end of July and beginning of August, 1917, which are referred to in the next chapter.

Regarded from the mathematical point of view to which reference was made in chap. IX, the curvature of the lines of flow and the consequent curvature of the isobars are part of the interplay of the inertia of the air and the forces which operate upon it, but in pursuance of our plan of selecting conditions which are amenable to treatment as special cases, we may note certain types or groupings of isobars which exhibit the characteristic of stability in the sense that they may last for days together without much change, and move bodily across the map. Pre-eminent among them, so far as our Islands are concerned, we have the group of isobars running from West to East across the Atlantic giving us a Westerly type of weather which was strikingly exemplified in the stormy winter of 1898–99, and represented by a band of Westerly current across the Atlantic, the high pressure on the South and the low pressure on the North. On the side of the low pressure there were considerable fluctuations which appeared as successive cyclonic depressions with great local intensity. With this type we include the South-Westerly type of weather in which the band of isobars runs from South West to North East.

In contrast with these we have the Easterly or North-Easterly types which are represented by bands of isobars running from East to West or, more often, from North East to South West with a high pressure to the North or North West. These also may last for weeks together and in that way be classed among the stable types[1].

In a sense it may be said that the Westerly type is oceanic and the Easterly type continental because the band of West to East isobars seldom penetrates far over the continent, it generally turns Northward on reaching the land, and the East to West band often turns Southward off the Western coast of France and uses its air-supply to feed the North East Trade wind.

Sometimes in our Islands we find ourselves in the region between a specimen of either of these types represented by a large quasi-permanent anticyclone to the North of us and another to the South of us. In that case we find the region between the two covered by a succession of rainy depressions. A good example will be found in the succession of maps in the month of July, 1918, particularly from the 15th to the 27th day of the month. This state of things might prove to be typical of the conditions for high latitudes for the band of West to East isobars if our maps extended far enough to the Northward to show fully the Northern anticyclonic region.

In any case the three sets of conditions mentioned, if we supplement them by the addition of the type of a quasi-permanent anticyclone directly over us, give a general idea of the classification of our weather as represented by locally persistent isobars and the lines of flow which accompany them.

[1] The travel of weather changes along the northern or southern side of a persistent band of high pressure is illustrated in *Weather of the British Coasts* (M.O. Publication, 230), chap. XI, § 9, 1918.

We must add the type represented by a band of isobars running from North to South which sometimes represents a quasi-permanent condition although Northerly winds are often only the transient accompaniment of the last stages of a passing cyclonic depression.

This will appear to be a digression but it has a bearing upon the relation of curvature to the permanence of type of air-motion in the atmosphere. There is the curvature of the earth's surface on the one hand and the curvature of the path of the air as it passes over the surface on the other hand; and both of them have to be considered in making a selection of possible permanent conditions underlying local and temporary fluctuations of weather. Perhaps the chart of mean isobars of the Northern hemisphere for the month should be regarded as the mean value from which the local conditions deviate from time to time, but the mean is made up from so many different types of map that it is better to classify the conspicuous types.

In dealing with the geostrophic wind we have taken the air as moving along a great circle but isobars drawn in a band for considerable distances along great circles converge[1], and a broad band arranged as the small circles of plane sections parallel to and on either side of a great circle would have different values for the latitude in different parts. The simplest form of distribution which we can regard as free from difficulties of that kind and the most stable form of atmospheric motion imaginable is that which is represented by a band of isobars from West to East or from East to West. A distribution of that kind represents part of a cap or series of rings of air rotating round the polar axis. The actual average distribution of isobars computed by Teisserenc de Bort[2] for the level of 4000 metres roughly represents such a distribution for the Northern hemisphere, and in the Southern hemisphere the distribution of pressure over the surface of the great Southern Ocean corresponds even more nearly with that ideal[3].

Within such a cap, revolving like a solid about the polar axis, there are no elements of instability. Any disturbances that arise must come from causes outside. Let us therefore consider a cap rotating as a solid round the pole as an ideal of stable atmospheric motion and the type to which the actual motion tends to revert when freed from the causes of disturbance to which it is subjected. The direction of motion may be either cyclonic or anticyclonic. The distribution of pressure for such a rotating cap would be as follows:

$$p_\phi = -\int_0^\phi V\rho\left(\omega + \frac{1}{2}\frac{V}{E}\right)\sin 2\phi\, d\phi \text{ for a cyclone and}$$

$$p_\phi = \int_0^\phi V\rho\left(\omega - \frac{1}{2}\frac{V}{E}\right)\sin 2\phi\, d\phi \text{ for an anticyclone,}$$

where V is the velocity of the wind at the equator.

[1] A suggestion as to the effect of the convergence in the northward component of air-motion is given in *Principia Atmospherica, loc. cit.* p. 88.

[2] 'Études sur la circulation générale de l'atmosphère,' *Ann. du B. C. M.*, 1885, 35–44. Hildebrandsson and Teisserenc de Bort, *Les Bases de la Météorologie Dynamique*, chap. IV.

[3] *National Antarctic Expedition*, 1901–1904. *Meteorology*, Part II, Royal Society, 1913.

Making an approximation, for the density, at 1250 g/m³ the distribution would be made up as follows:

TABLE I. VELOCITIES AND CORRESPONDING VALUES OF THE PRESSURE AND ITS GRADIENT FOR DIFFERENT LATITUDES IN A HEMISPHERICAL CAP OF AIR REVOLVING LIKE A SOLID ABOUT THE POLAR AXIS.

		Pressure		Gradient			
Latitude °	Velocity m/s	Cyclone mb.	Anticyclone mb.	Geo-strophic component mb./deg.	Cyclo-strophic component mb./deg.	Total for cyclonic system mb./deg.	Total for anticyclonic system mb./deg.
0	15	1054	967	0	0	0	0
10	14·8	1051	970	0·52	·01	0·53	0·51
20	14·1	1044	977	0·98	·015	1·00	0·97
30	13·0	1032	989	1·32	·02	1·34	1·30
40	11·5	1018	1003	1·50	·02	1·52	1·48
50	9·7	1002	1017	1·50	·02	1·52	1·48
60	7·5	988	1031	1·32	·02	1·34	1·30
70	5·2	976	1043	0·98	·015	1·00	0·97
80	2·6	968	1050	0·52	·01	0·53	0·51
90	0	966	1053	0	0	0	0

Such a cap is obviously not fully represented by the distribution of winds over the earth's surface. There are Westerly winds in middle latitudes and in the Southern hemisphere they extend round the earth, but not in the Northern hemisphere; and in neither do they extend to latitudes below 35°. Sometimes it would appear as though the winds in the Southern hemisphere might constitute a cap, rotating from West to East, over a lower layer rotating from East to West or over the Antarctic continent which occupies a great part of the room that would belong to the lower rotating anticyclonic cap. And in the Northern hemisphere in the region of the British Isles there is sometimes, as already mentioned, a great anticyclone to the North of our Islands covering Greenland, Iceland and the North of Scandinavia suggesting a portion of an anticyclonic cap, while the pressure to the South of us is arranged in West East lines and corresponds with that of a portion of a cyclonic cap in middle latitudes South of the anticyclonic cap. Between the two is a belt bounded by isobars of the same designation between which local cyclones are shown[1]. The conditions in the Northern hemisphere are singularly unpropitious for a regular anticyclonic circulation of the lower layers of the air round the pole because the land mass of Greenland ten thousand feet high stretching from beyond the eightieth to the sixtieth parallel effectually blocks the way. The Easterly wind of the lower atmosphere cannot go over it and must be diverted round it.

It is generally allowed that the maintenance unchanged of the length of the period of revolution of the earth is conclusive evidence against an average

[1] Shaw, 'On the General Circulation of the Atmosphere in Middle and Higher Latitudes,' *Proc. Roy. Soc.*, vol. LXXIV, p. 20, 1904, and *National Antarctic Expedition*, 1901–1904, *Meteorology*, Part I, Royal Society, 1908, p. xiii.

rotation of the whole atmosphere in one direction. Hence the existence, in one part, of a belt with one kind of rotation implies the existence of its opposite equivalent somewhere else.

We have given attention to the idea of a cap, or a belt of air forming part of a cap round the globe, because it may have some bearing upon the situations which we have to consider although no complete cap or belt is shown. There is nothing at all in the way of mutual influence to bind one part of a cap to another part on the other side of the world; each part must be separately maintained by an appropriate environment. Just as a small sector of a rainbow will exhibit all the essential properties of the complete arc so in atmospheric motion a sector of a belt may exhibit all the essential properties of a complete belt. Wherever a group of isobars is formed which, so far as they extend, would correspond with a portion of such a cap, the air within the portion will have the same kind of stability, and be subject to the same conditions as the cap would satisfy, although it is maintained in its successive positions by a different environment. It follows that the bands of air-flow which maintain themselves sometimes for days, sometimes even for weeks, may have the kind of stability that belongs to a rotating cap and be quite as important elements of the atmospheric circulation as anticyclones or cyclonic depressions and have the same kind of permanence if their environment permits.

For these bands of limited extent, which may be treated as sectors of belts, the effect of curvature is small. According to the formula of p. 1, the ratio of the cyclostrophic component to the geostrophic component is one-half of the ratio of the equatorial velocity of the cap to the equatorial velocity of the earth's surface. In the case represented in Table I it amounts to 1·6 per cent.

The other type of atmospheric motion which claims attention is rotation round a centre in a circle of much smaller radius than 40°, indeed it may be taken as being from 10° down to 1°, or even less in the case of tornadoes and water-spouts. There is evident stability in motion of this character because beginning with examples of whirls lasting for some seconds there is apparently an uninterrupted sequence by way of revolving sandstorms or dust-devils, tornadoes, or whirlwinds, to tropical revolving storms and large cyclonic areas with radii of 10° or more. The only limit of the series is a revolving air-cap covering a hemisphere or a large part of it. And just as a belt of West wind or a belt of East wind may lie over these Islands for weeks so the other type of quasi-permanent atmospheric motion, which has always been thought of as a column of air in continuous revolution, may preserve its identity for days or weeks. Through the kindness of Professor McAdie of Blue Hill Observatory, Harvard University, we are enabled to give two notable examples.

The first is that of a tropical revolving storm which started on a Westerly track towards the Philippine Islands (where visitations of that kind are known as "Baguios"), turned round towards the North and North East, crossed the Pacific Ocean and after some vagaries on the North American continent continued its journey Eastward and crossed the Atlantic in the usual track of cyclonic depressions over that ocean. The whole journey lasted from

20 November 1895 to 22 January 1896. The second is a cyclonic depression of October 1913, in the outer region of which the tornado was formed which caused so much destruction in South Wales on the twenty-seventh of that month[1]. The track of the main depression shows an anomalous path from Canada to the North of the British Isles. The tracks of the centres of these depressions are shown upon the map which forms the frontispiece of this part.

To these notable examples has been added the long track of a cyclonic depression which was figured in the Meteorological Office chart of the North Atlantic and Mediterranean for August 1904[2]. The cyclone was first noted on 3 August 1899 in that part of the North Atlantic Ocean where West Indian hurricanes often take their rise. It moved Westward to the West Indies, skirted the coast of Florida and turned Eastward over the Gulf Stream. After some hesitation about latitude 40° W. it made for the mouth of the English Channel and, missing that, crossed to the Mediterranean where it lost itself on 9 September after a life of thirty-eight days.

Thus out of the kaleidoscopic features of the circulation of air in temperate latitudes two definite states sort themselves each having its own stability. The first represents air moving like a portion of a belt round an axis through the earth's centre. It is dependent upon the earth's spin and the geostrophic component of the gradient is the important feature; the curvature of the isobars is of small importance. The second represents air rotating round a point not very far away: it is dependent upon the local spin, and the curvature of the isobars with the corresponding cyclostrophic component of the gradient is the dominant consideration.

In reality of course both components are operative in all cases except at the equator where the geostrophic component is zero. Only the geostrophic component depends upon the latitude. The numerical importance of the cyclostrophic component increases rapidly with the velocity of the spin and is paramount for rotational systems of small radius. The following table shows the velocities in different latitudes for which the two components are equal when the radius is 100 kilometres.

Latitude in degrees	90	80	70	60	50	40	30	20	10	0
Velocity with 100 k. radius for equal components (in metres per second)	14·6	14·3	13·7	12·6	11·2	9·4	7·3	5·0	2·5	0

Just as in the case of the belts of air revolving about a diameter of the earth so in the case of a mass of air in rotation with a small angular radius, there is no influence of one part of the whirl upon another part in another sector which holds them together; each sector, or truncated part of it, must satisfy independently the conditions which are applicable to any and every part of the whole whirl; and this it can do, without the other sectors, provided that its environment is suitable for its persistence. Experience of maps would

[1] *Geophysical Memoirs*, No. 11. M.O. Publication, No. 220a.

[2] M. O. Publication, No. 149.

seem to indicate that the complete circular form is not necessary for the persistence of a part. We may make use of this principle later on but it is convenient for the present to deal with those cases in which the circular form is complete. We shall therefore proceed with our study of curved isobars by considering the case of a short circular column or disc of air which is in rotation about a centre.

We shall suppose that the disc of air rotates like a solid, in which case the angular velocity of each portion of the disc will be the same. Subsequently we can consider whether that assumption is reasonable from the meteorological point of view. We may take the common angular velocity to be ζ. The linear velocity v at any point distant r from the centre will be $r\zeta$. Its direction will be at right angles to r. If the motion belongs to a cyclone, in the Northern hemisphere it will be counter-clockwise. For the sake of brevity we will identify such a distribution of winds as a normal cyclone. The name is not a very happy one because there is no evidence either for or against its representing the average conditions of motion of a cyclone in the upper air. At the surface there is always some incurvature of the wind with respect to the circular path round the centre.

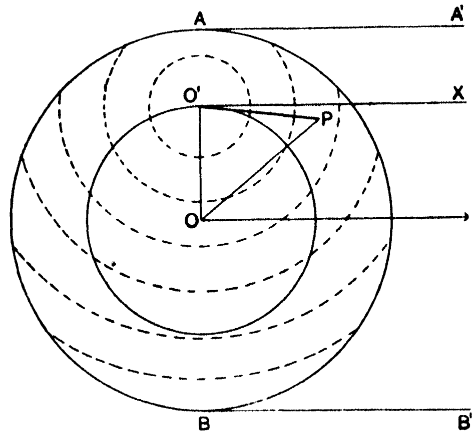


FIG. 1. Combination of rotation and translation.

The characteristic feature of our cyclonic depressions is that they travel across the map. The velocity of travel is very varied but when the depression is represented by circular isobars it generally has a rapid speed of travel. A velocity of 20 m/s for the centre of a cyclonic depression is large but not unknown, a velocity of less than 10 m/s may be regarded as smaller than the average. A tropical revolving storm usually travels at about 4 m/s.

If the rotating disc forming a horizontal section of the normal cyclone travels bodily without altering its shape or velocity in rotation, in order to obtain the scheme of actual velocities the velocity of translation V must be superposed upon the velocity in rotation of each part of the disc. We have thus to deal with the combination of rotational motion with translational motion. That is the subject of certain well-known propositions of which we require the following.

1. A disc which is rotating like a solid round a point with angular velocity ζ and is travelling in its own plane with a velocity of translation V will have a point O' *instantaneously at rest*, where OO' is equal to V/ζ and O' is on the radius of the disc drawn transverse to the line of travel and to the left; the disc will be *instantaneously in rotation round the point* O' with the angular velocity ζ.

This proposition depends upon the geometrical addition:

the vector $O'O$ + the vector OP = the vector $O'P$.

If the vector in each case is the velocity proportional to and at right angles to the lines named the conclusion follows directly from the parallelogram of velocities.

2. The centre of instantaneous rotation O' will "travel" along a line parallel to the path of the centre of the disc with the same velocity of travel as the centre of the disc. The word travel is conventional in this statement. Nothing really travels along the line $O'X$. New systems of rotation are developed with successive points on the line as centres.

3. The actual paths of the particles of the disc will be the series of curves traced out by the several points of the disc when the circle with radius $O'O$ rolls along the line $O'X$.

These curves are well known in geometry. The path of O will be a straight line, the paths of the points in the circumference of the circle with $O'O$ as radius will be cycloids and of other points trochoids. Those points which are further from O than O' will form looped curves: those within that limit form curves in the shape of long sea-waves. The curves are represented later as figure 7.

All this geometry may be summed up by saying that rotation like a solid combined with translation can be represented by consecutive instantaneous rotation round a travelling centre at a distance V/ζ on the left of the path of the permanent centre of rotation; and the actual paths are the curves formed by points attached to a circle which rolls along the line of instantaneous centres and has for its radius the distance between the permanent and the instantaneous centres of rotation.

The geometry of the combination of rotary motion with translation is difficult for those who are not familiar with it, but obviously it has to be dealt with in all cases of travelling circular motion with which for sixty years now we have been accustomed to classify cyclonic depressions.

Some readers may be unwilling to regard instantaneous rotation in a circle as a fair representation of the motion of air in a cyclonic depression. They will point out that the lines of flow of air are spirals meeting in the centre of the cyclone towards which all the air directs its motion. The difference is less important than would appear at first sight. In order to find out what its practical effect would be a machine for drawing the actual paths, with given rate of travel and given angle of incurvature, was constructed by the Scientific Instrument Company of Cambridge[1]. The effect of the incurvature was not that any paths led directly to the centre but that the loops on the curves had a curious tilt to the left. The subject was treated analytically by the late Professor W. H. H. Hudson[2].

It will be noticed that the circles showing the instantaneous rotation round the point O' are incomplete because the real rotating disc is not

[1] *Life-History of Surface Air-Currents*, p. 100. [2] 'Anemoids,' *B.A. Report*, 1906, p. 483.

sufficiently extensive. That is of no consequence. We are quite accustomed to it in the case of a cart-wheel for which the centre of instantaneous rotation is the point which touches the ground and, in consequence, is instantaneously at rest. No inconvenience arises: the arcs of circles representing the instantaneous motion of a cart-wheel are merely aids to comprehending the situation; so are the circles round the centre in fig. 1: they have no permanent existence yet they represent the lines of flow of the actual winds of a normal cyclone.

On the other hand, we may note that the actual disc in its rotation will use the whole space between the lines AA', BB'; so that, in order to represent a complete normal cyclone, there must be something besides the complete circles of instantaneous rotation, namely the parts of the instantaneous circles required to fill the circular boundary of the disc. We can represent the component and resultant motions by fig. 2, the first diagram of which repre-

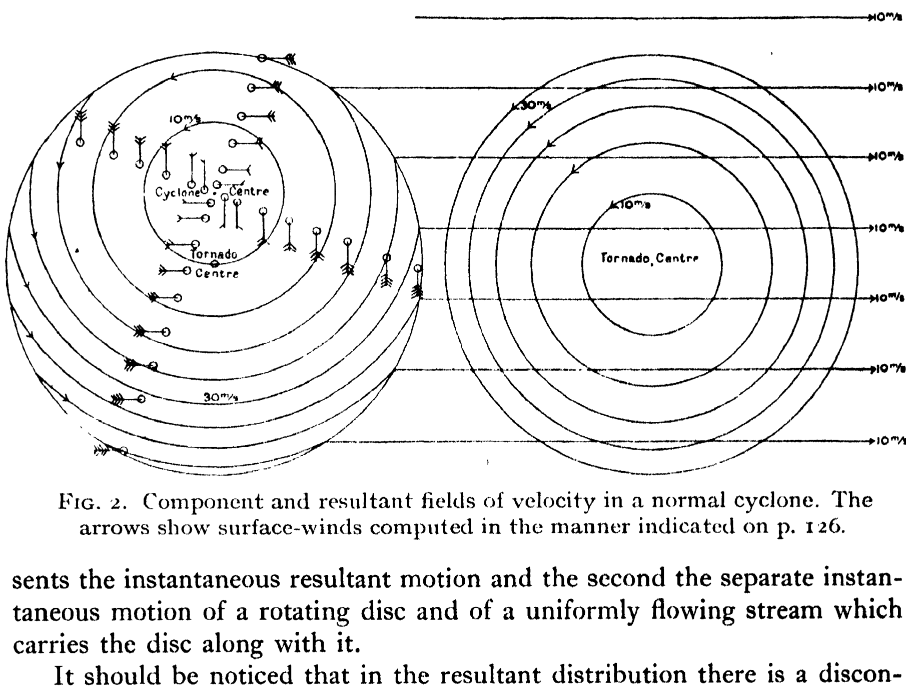


FIG. 2. Component and resultant fields of velocity in a normal cyclone. The arrows show surface-winds computed in the manner indicated on p. 126.

sents the instantaneous resultant motion and the second the separate instantaneous motion of a rotating disc and of a uniformly flowing stream which carries the disc along with it.

It should be noticed that in the resultant distribution there is a discontinuity of velocity at the junction of the disc and the stream. In the diagram the finite angle seems to be between the flow outside and the flow inside the disc, making it appear that fluid crosses the boundary of the disc; that, of course, is not the case. The component in the direction of flow is common to both and the relative motion is the rotation of the boundary of the circle. The discontinuity would in actual cases cause eddies and the nature of the field of flow in the immediate neighbourhood of the rotating disc requires some adjustment on that account which is not yet made out.

Thus, in a normal cyclone there are two centres, one the centre of instantaneous rotation, the point instantaneously at rest which we call the *kinematic*

centre, and the other the proper centre of the rotating disc which we call the *tornado centre*. They lie in a line at right angles to the path of the cyclone and are separated from each other by a distance V/ζ, where V is the velocity of travel and ζ is the angular velocity of rotation of the disc.

On a map representing a normal cyclone the observed winds would give the resultant velocities for those points at which the stations happened to be placed. A complete system of observations would enable us to construct the diagram of instantaneous lines of flow represented in fig. 2. It must be noted that this diagram gives no means of identifying the tornado centre. It is, in fact, the point where the direction and velocity of the wind are the same as the direction and velocity of the travel of the whole disc, but unless we can determine that velocity with precision or determine the ratio V/ζ there is nothing to point out its actual position and certainly no student of weather-maps, unless he had been previously forewarned, would suspect that the permanent centre of rotation of the disc was at a point which gave no indication of its presence by any obvious peculiarity of the winds in the neighbourhood.

The next question for consideration is the fitting of a system of isobars to the lines of flow of the normal cyclone. The answer is that if we consider the travel of the cyclone to take place along a horizontal plane and neglect the curvature of the earth's surface, the variation of the geostrophic relation with the latitude due to the variation in $\sin\phi$, and also the small variations of density that occur during the travel of the air in the horizontal layer, the appropriate system of isobars will consist of a field of circular isobars, corresponding with the rotation of the disc, embedded in a field of straight isobars corresponding with the velocity of translation, but that the centre of the circular isobars will coincide neither with the centre of the rotating disc, the tornado centre, nor with the centre of instantaneous rotation, the kinematic centre. It will be at a point on the line joining those two centres at a distance from the kinematic centre equal to $V/(2\omega\sin\phi + \zeta)$.

This will constitute a third centre for the normal cyclone. It will be at the centre of isobars as drawn on the map and therefore quite easily identified. We call it the *dynamic centre*. Since cyclones were mapped it has always been regarded as the centre of the cyclone, but clearly it is not unique in that. Our ideas about the relation of the features of a normal cyclone to its centre will not be complete unless we recognise that the permanent rotation is centred at another point, the tornado centre, and the instantaneous winds are centred about a third point, the kinematic centre.

The demonstration of the position of the dynamic centre, the centre of isobars, with reference to the tornado centre, the centre of permanent rotation of the disc, follows simply from the combination of the field of pressure representing the velocity of travel according to the geostrophic law and the field of pressure representing the rotation of the disc with its cyclostrophic and geostrophic components.

Taking the centre of the rotating disc as origin with the axis of x to the East and that of y to the North, since for uniform Eastward motion the pressure

diminishes uniformly to the North at the rate $2\omega\rho V \sin\phi$ the geostrophic field of pressure for a velocity of translation V to the East will be

$$p' - p_0' = -2\rho\omega V y \sin\phi \qquad \text{......(1)},$$

where p' is the pressure at any point, p_0' the pressure at any point on the axis of x. The equation to the circular field which would balance the rotation of the disc round a stationary centre at the origin is

$$p - p_0 = \tfrac{1}{2}\rho\zeta\,(2\omega\sin\phi + \zeta)\,(x^2 + y^2) \qquad \text{......(2)}.$$

Equation (2) is the direct integration of the gradient equation

$$dp/dr = \rho\,(2\omega v \sin\phi + v^2 \cot r/E).$$

Neglecting the curvature of the earth the second term becomes v^2/r, and v is equal to $r\zeta$; hence

$$r^{-1} dp/dr = \rho\zeta\,(2\omega\sin\phi + \zeta),$$

whence, since ρ and ϕ are taken as constant

$$p - p_0 = \tfrac{1}{2}\rho\zeta\,(2\omega\sin\phi + \zeta)\, r^2,$$

and $$r^2 = x^2 + y^2.$$

Combining the two fields by adding equations (1) and (2) and writing P for the resultant pressure, we get for the equation of the resultant field

$$P - P_0 = \tfrac{1}{2}\rho\zeta\,(2\omega\sin\phi + \zeta)\,(x^2 + y^2) - 2\rho\omega V y \sin\phi \qquad \text{......(3)}.$$

This equation represents a circular field of pressure round the centre

$$x = 0,\ y = \frac{V 2\omega\sin\phi}{\zeta\,(2\omega\sin\phi + \zeta)},$$

and the pressure P_0 is at the centre instead of the origin. The distance of the dynamic centre from the tornado centre which was chosen as the origin is

$$\frac{V}{\zeta} \times \frac{2\omega\sin\phi}{2\omega\sin\phi + \zeta},$$

and the distance of the kinematic centre from the tornado centre is V/ζ. Hence the distance of the dynamic centre from the kinematic centre is the difference, that is $V/(2\omega\sin\phi + \zeta)$.

It follows that a system of circular isobars embedded in a field of straight isobars corresponding with the velocity of travel, having its centre at a properly selected point, will provide the field of pressure necessary for the disc to go on rotating. The component and resultant fields of pressure are represented in fig. 3.

That the centre of isobars is not coincident with either of the centres of rotation is a peculiarity of atmospheric motion arising from the fact that in consequence of the rotation of the earth a field of pressure is required to balance what appears in our reasoning as rectilinear motion in a horizontal plane rotating uniformly with an angular velocity $\omega\sin\phi$.

These conclusions enable us to approach the true position with regard to the relation of winds to curved isobars which we were unable to deal with previously owing to our ignorance of the curvature of the path. For a travelling system of isobars in the form of circles the winds computed according to the gradient equation of page 1 would be a system of winds arranged instantaneously in circles round the kinematic centre distant $V/(2\omega \sin \phi + \zeta)$ from, and on the left-hand side of, the path of the centre of isobars. It would indicate the existence of a tornado centre of permanent rotation at a distance

$$\frac{V}{\zeta} \times \frac{2\omega \sin \phi}{2\omega \sin \phi + \zeta}$$

on the right of the path of the centre of isobars.

The next question is what resemblance the results of this calculation bear to reality as represented on charts of pressure and wind. We cannot make a

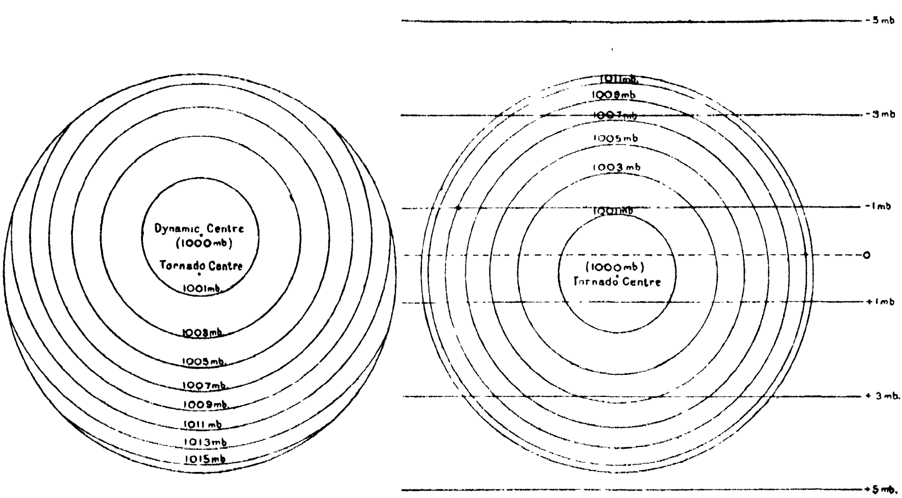


FIG. 3. Component and resultant fields of pressure for a normal cyclone.

direct comparison without a further application of theory because what we have dealt with relates to a normal cyclone in the free atmosphere and our weather-maps represent the pressure and wind at the surface. The representation of the pressure will serve for the surface as well as for the free atmosphere, but the winds at the surface will be affected by the eddy-motion due to the friction of the ground. We can make an allowance for the effect, according to Taylor's theory as in fig. 2, by assuming a deviation of the wind from the direction of the isobars of 20° and a corresponding reduction of the velocity to two-thirds of that corresponding with the gradient in the free air. With this further application our theory is complete, except that we have not dealt with the discontinuities in velocity and pressure-gradient at the boundary of the rotating disc. In order to test the conclusions a theoretical map has been constructed showing a normal cyclone of 10 m/s at a distance of 100 kilometres

which travels in a broad westerly current at the rate of 20 m/s. The isobars in which the normal cyclone is formed are drawn curved but that should make little difference. The region of discontinuity has been treated by bending the isobars in their course on either side of the boundary of the revolving disc. The map is reproduced in fig. 6.

For comparison with this theoretical map we reproduce in fig. 4 the weather-map for 18 h of September 10, 1903, representing an actual cyclonic storm of that date.

Let us note the comparison. The general similarity of the two maps is obvious, perhaps it has been made improperly so by the smoothing of the discontinuity and by drawing an arbitrary isobar outside the region of revolution, but it is confirmed by the agreement between the actual velocity of travel of the storm which is given in the *Life-History* as 16 m/s and the geostrophic velocity deduced from the isobars of its path which is 17 m/s. The scale of winds which defines the angular velocity in the theoretical cyclone, namely 10 m/s at a radius of 100 kilometres, is perhaps too large, as force 11 is shown in the outer rings of the theoretical cyclone and is an unusual figure for a surface-wind. Little change, however, in the general character would be introduced by adjusting the ratio for a more suitable value of the wind-velocity in the outer rings.

There are two features of agreement which are very striking, one is the existence in the real case of close isobars in the form of incomplete circular arcs which are required to complete the mapping of the revolving disc in the theoretical map. They will be recognised as characteristic of small cyclones which travel rapidly. The other is the peculiarity of the wind indicated as passing outward from the innermost isobar of the real cyclone. A wind with similar disobedience to the run of the isobars on the Northern side of the centre is shown in many other actual maps of rapid travelling storms. There are several included in the carefully drawn maps of the *Life-History*. We have been accustomed to regard them merely as unimportant irregularities to be expected from the light winds which are found near the centre of a cyclone, but if the theoretical map is correct so also are these hitherto irregular winds. They mark the rotation of the winds round a centre not coincident with the centre of isobars but on the left of its path, and that conclusion is confirmed by our previous inference that light winds are not excluded from the influence of the distribution of pressure but show closer agreement with it than stronger winds. Another interesting feature of similarity is the incurvature shown by the winds in the rear of the storm as compared with the stricter agreement with the run of the isobars in the front. These considerations lead us to accept the conclusion to be drawn from the conditions of the normal cyclone, namely that the wind *calculated from the gradient by the full formula, using the curvature of the isobars, gives the true wind in the free air not at the point at which the gradient is taken but at a point distant from it along a line at right angles to the path and on the left of it by the amount* $V/(2\omega \sin \phi + \zeta)$. For the particular cyclone represented in fig. 5, the distance is about 50 kilometres. It increases

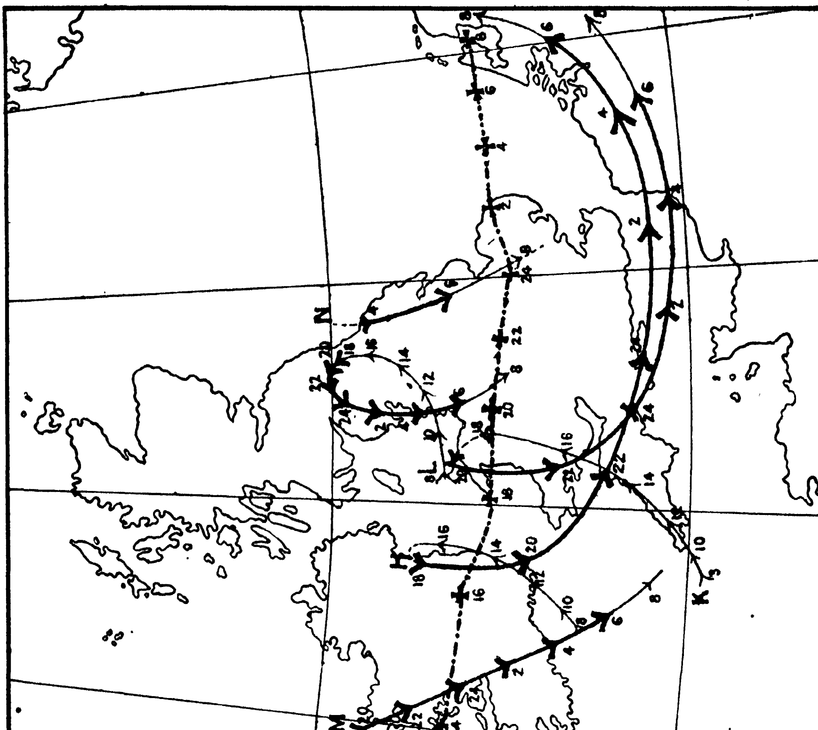


FIG. 5. Trajectories, or actual Paths of Air.

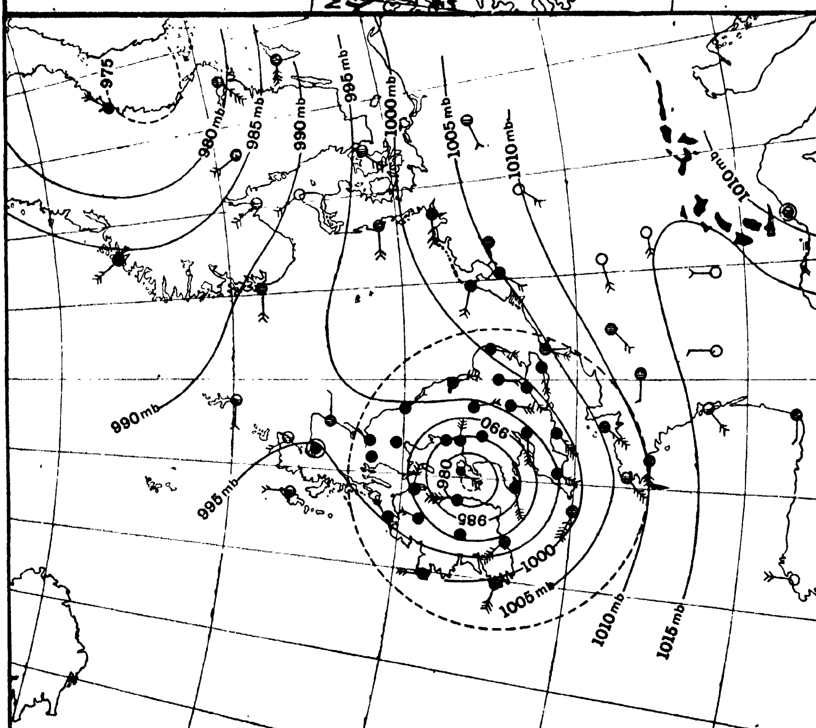


FIG. 4. Map of cyclone of September 10, 1903, 18 h.

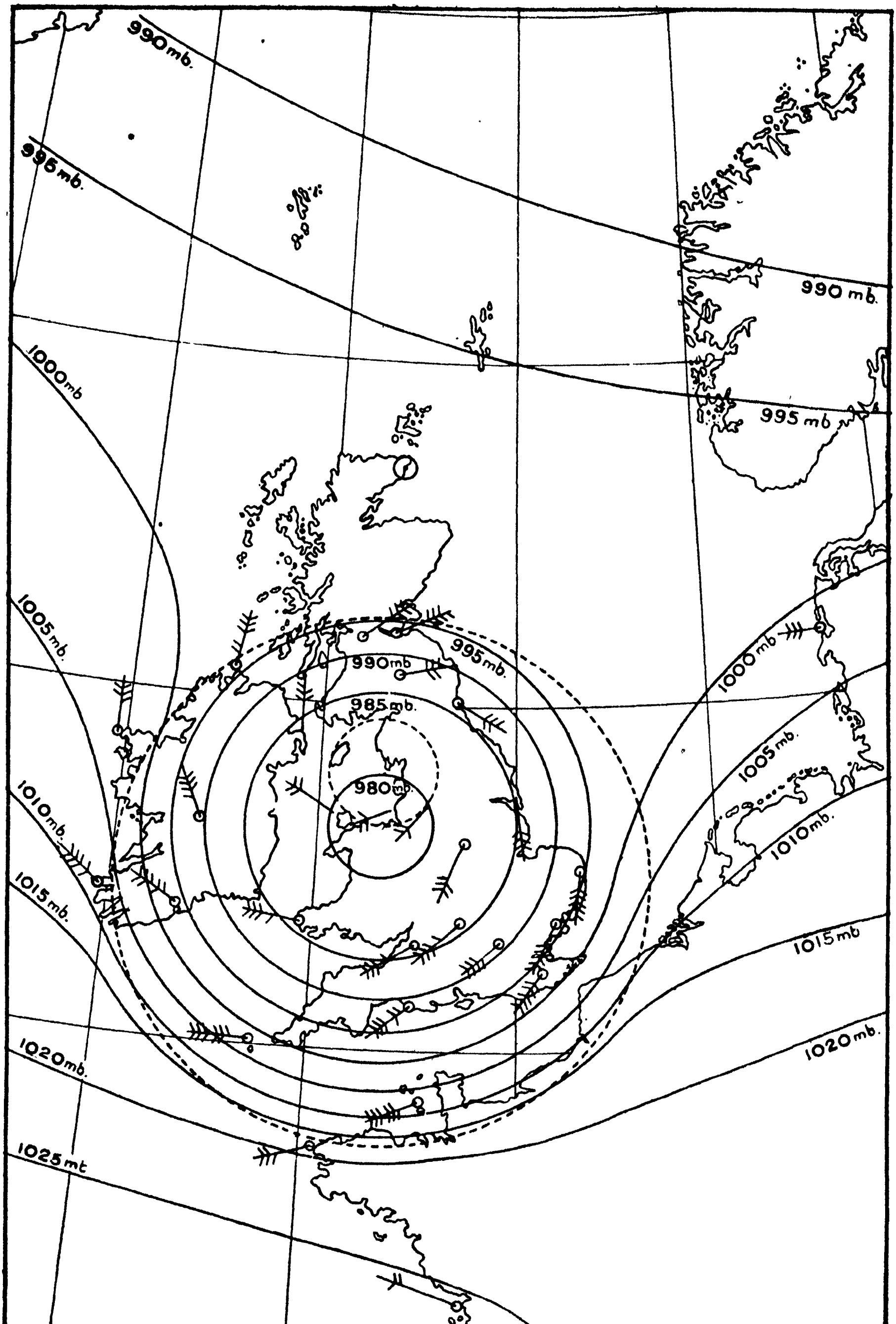


FIG. 6. Normal cyclone, of 10 m/s at 100 kilometres, in a westerly current of 20 m/s. The larger dotted circle shows the boundary of the revolving fluid; the smaller, the position of the kinematic centre of instantaneous rotation.

with the velocity of translation V, and when ζ is equal to $\omega \sin \phi$ and the air in consequence forms part of a cap rotating as a solid round the pole it is equal to $V/(3\omega \sin \phi)$.

And more striking still is perhaps the agreement to be found between the actual paths of the air in the cyclone of September 10–11, 1903, constructed many years ago without reference to any theory whatever from successive hourly maps for the *Life-History*, for which the original diagrams are reproduced in fig. 5, and the calculated paths of the air in the normal cyclone as shown in fig. 7.

Corresponding agreement is to be found in the case of the circular storm of March 24–25, 1902, which is also figured in the *Life-History*, but in that

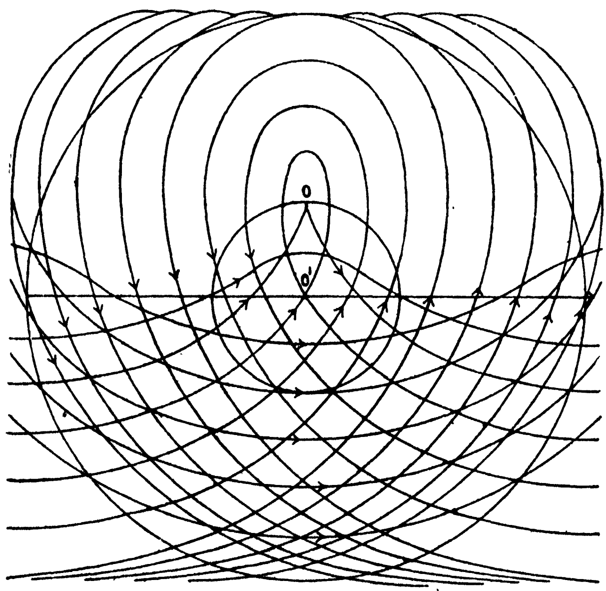


FIG. 7. Paths of air for normal cyclone.

case the isobars which governed the rate of travel are found in the rear of the storm not in the front. It must be remarked here that what has been said about the travel of circular storms cannot be applied to the cyclonic depressions which often succeed one another in a belt between two isobars one on the North and the other on the South, as in the case of the series of depressions of July, 1918, and in that of the slowly moving depression of November 11–13, 1901, also figured in the *Life-History*. The environment of the cyclone is quite different in these cases and the cause of the travel must be different. Cordeiro[1] has pointed out that the gyroscopic effect of a revolving column on a rotating earth necessitates a motion of the column in order to keep its axis in the progressive vertical, so that if a cyclone is to persist it must travel

[1] F. J. B. Cordeiro, *The Atmosphere*, New York, Spon and Chamberlain. London, E. and F. N. Spon. Ltd.. 1910.

even if it is formed in a body of calm air, but this subject belongs more properly to the next chapter. In this chapter we have assumed the motion to be over a plane rotating surface and the examples are taken from cyclones with rapid motion of translation.

There is another aspect of the combination of a circular field of pressure with a linear field to produce the distribution of pressure which has been shown to be necessary for the travel of a normal cyclone which is interesting from the point of view of travelling groups of curved isobars. For the reasons which we give below it appears that if circumstances were so arranged that a linear field of force with gradient s suddenly came into operation upon a stationary normal cyclone it would forthwith produce the field of force appropriate for a normal cyclone travelling with the velocity V, where V is equal to $s/(\zeta\rho)$, along a line at right angles to the superposed gradient and *from left to right* of a person facing the high pressure. And, since the travelling normal cyclone has a distribution of velocity round the instantaneous centre indistinguishable from that of the normal cyclone round its tornado centre, it follows that the sudden superposition of a gradient would *ipso facto* transform the stationary cyclone into a cyclone travelling with the assigned velocity.

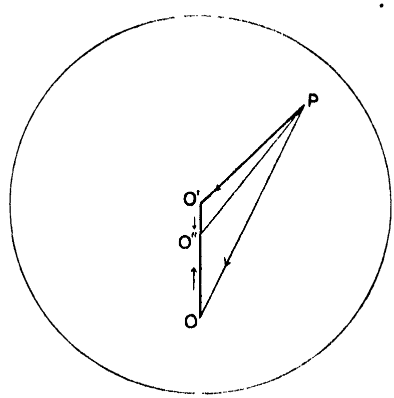


FIG. 8. Three centres of normal cyclone.

In *Geophysical Memoirs*, No. 12, this conclusion was reached from the consideration of a proposition set out by Gold[1] to the effect that in a travelling cyclone which consists of rings of fluid in instantaneous rotation about a moving axis the relation between X, the radius of curvature of the path, and r, the radius of the instantaneous circle, is given by the equation
$$X = r/(1 + V \sin \alpha/v),$$
where α is the angle which the radius makes with the line of path of the kinematic centre. But the conclusion follows very easily from the properties of the three centres of the normal cyclone, fig. 8.

Let O' be the centre of the stationary normal cyclone, O the tornado centre for a velocity of travel V, so that $OO' = V/\zeta$. Let P be any point of the rotating air. The acceleration at P will be $\zeta(2\omega \sin\phi + \zeta)\,OP$. Suppose a field of uniform gradient s (equal to $V\rho\zeta$) to be superposed upon the system in the direction $O'O$; the acceleration of P corresponding therewith is $V\zeta$. It will be represented by a line $O'O''$ where O'' is defined by the condition

$$O'O'' : OP :: V\zeta : \zeta(2\omega \sin\phi + \zeta)\,OP.$$

Hence $$O'O'' = V/(2\omega \sin\phi + \zeta).$$

[1] *Barometric Gradient and Wind Force*, M. O. Publication, 190, p. 43.

Thus O'' is also the dynamic centre of the normal cyclone with tornado centre O, travelling with the velocity V, and the resultant acceleration $\zeta(2\omega \sin \phi + \zeta) O''P$ is in every respect the same as the combination of an acceleration $\zeta(2\omega \sin \phi + \zeta) OP$ towards the tornado centre O with an acceleration $V/(2\omega \sin \phi)$ proportional to OO'' across the line of travel from right to left. That is to say, the resulting gradient or distribution of pressure is the same in direction and magnitude as that required for a travelling normal cyclone centred at O. The difference between the curvature of the path and the curvature of the circle of instantaneous motion is expressed by a linear field of pressure which tends to push the rotating air towards O. The effect of the push is not to displace the system in the direction of the push but to make the instantaneous centre of rotation travel from left to right across the line of the push carrying, of course, the rotating system with it.

At the moment of the superposition of the new transverse gradient there is nothing (except the extent of the area affected) to distinguish the motion of rotation round O' from an identical motion of rotation round O combined with a translation V: they are simply two aspects of the same field of motion. Consequently when the new field is superposed all the forces will be accommodated if instead of continuing the original rotation round the point O', another aspect of the same motion is continued, namely, rotation round O and travel with the velocity V.

If instead of a finite gradient being suddenly superposed the superposition was gradual as, for example, by the passing overhead of a system of isobars belonging to the region of the stratosphere, the development of the corresponding travel of the normal cyclone would be similarly gradual. If we pursued the matter further we should have to recognise that only a part of the stationary cyclone forms complete circles round the point O; the new permanent rotation would be lop-sided. Also the stationary cyclone has to be imagined in calm air, and in the new conditions it is through the surrounding air that it would have to make its way and we can make no estimate of the reactions that would ensue. The new gradient, if it extended beyond the area of the cyclone, would not help matters as regards the environment; for the balancing motion corresponding therewith is in the opposite direction from that in which the cyclone has to move.

What modification in the subsequent motion these considerations would introduce we are, therefore, at present unable to say, but without that further development the result obtained is of considerable interest to us in our pursuit of an answer to the question of the relation of the wind to the distribution of pressure. That the wind should always be regarded as balancing the gradient is a hard saying for many meteorologists. It has even been said that the assumption simply ignores the causes through whose operation the changes which we wish to study are brought about. We may attempt to devise circumstances under which finite changes of pressure would conceivably come into operation so quickly that the theoretical adjustment could not have been approached. The reader must be content to judge whether the conclusions

which have been drawn from the assumption throw sufficient light on some of the hidden atmospheric processes to make it worth the risk. From the results of our work it seems possible that as a general rule in the free atmosphere in ordinary circumstances the disturbances of the balance are not large enough to interfere with the conclusions to be drawn from it, but there may be in special localities singular points or lines, points or lines of convergence or divergence, and therefore of convection, to which we cannot apply the assumption of a balance between the field of pressure and the field of velocity.

The last proposition enables us to get some insight into the effect of a difference between the actual field of pressure and the field required for the balance, which may be generalised by saying that with curved isobars the uncompensated part of the field will express itself in the travel of the group of isobars. The only case that we have dealt with algebraically is that in which the uncompensated field is uniform, but we may suppose that in general the effects will be similar when the condition is not satisfied; and here we may with advantage revert to the principle that each sector which preserves its identity is subject to the same conditions as a complete rotating system. If we find on the map a persistent group of isobars consisting of segments of circles and if we can by any means identify the instantaneous centre of its winds, the difference between the actual field and the field appropriate to permanent rotation round the instantaneous centre will be expressed by a proper motion of the group of isobars. The identification of the kinematic centre must depend upon further study of the field of motion in actual cases.

Our consideration has been limited by the restriction to motion over a plane surface, we have not yet been able to extend it to a spherical surface. It would be interesting to know what the effect of a specified uncompensated field would be upon the motion of a rotating cap of air such as those which we considered in the earlier pages of this chapter, but we are not aware of any investigation which answers this question.

A further matter of interest is the travel of anticyclones regarded from the standpoint here adopted for that of cyclones, but that must be left for the reader's own reflexion.

CHAPTER XI

Revolving Fluid in the Atmosphere

THE literature of dynamical meteorology is largely concerned with the idea of revolving fluid as represented perhaps primarily in tropical revolving storms and in other whirls on a smaller scale and then by a process of analogy in the phenomena of the cyclones and anticyclones which had taken an established place as the primary features of the weather-maps of middle latitudes. In introducing the name *anticyclone* for the regions of closed isobars surrounding a centre of high pressure Sir Francis Galton[1] used the following words: "Most meteorologists are agreed that a circumscribed area of barometric depression is a locus of light ascending currents and therefore of an indraught of surface winds which create a retrograde whirl (in our hemisphere). Consequently we ought to admit that a similar area of barometric elevation is a locus of dense descending current, and therefore of a dispersion of a cold dry atmosphere plunging from the higher regions upon the surface of the earth, which, flowing away radially on all sides, becomes at length imbued with a lateral motion due to the above mentioned cause, though acting in a different manner and in opposite directions."

Based upon the representation of the process which these words convey there gradually grew the conception on the one hand of the central area of a cyclone on the map as a centre of centripetal motion, a focus of attraction for the surrounding air and the general idea of a cyclone as a region of ascending warm air producing rain or snow; round the central region the air moves inward with a counter-clockwise motion in spiral curves. On the other hand the conception of the central area of an anticyclone is of a centre of centrifugal motion, a region of repulsion; the general area of an anticyclone as a region of descending cold air which moves with a clockwise motion spirally outwards. The conventional representation of cyclones and anticyclones included the instantaneous lines of flow as a series of double spiral or reversed S-shaped curves leading from centres of high pressure to centres of low pressure. We have explained elsewhere the objections which may be urged against these conceptions of the physical nature of cyclones and anticyclones. In the *Life-History of Surface Air-Currents* the paths of air over the surface were shown to be quite different in actual shape from the instantaneous spirals and to include motion from low pressure to high pressure, despite the instantaneous incurvature towards the "low," in consequence of the travel of the isobars. And in previous sections of this work we have given reasons drawn from the conditions of thermal convection which controvert the idea of a descending

[1] *Proc. Roy. Soc.*, vol. XII, p. 385, 1862–3.

column of air, assumed to be cold, in the central region of an anticyclone in direct relation with an ascending column of air, assumed to be warm, in the central region of a cyclone.

Here we wish to remind the reader that the idea has been found singularly sterile as a means of developing our knowledge of the physical processes which are expressed by the weather which we experience. The grouping of the phenomena with reference to the centre of isobars of travelling cyclones has proved ineffective for this purpose. There was no symmetry with respect to the centre for any of the meteorological elements with the exception of pressure and, to a certain extent, of the winds. The difficulty lay in the fact that the feature of the phenomena for which it was necessary to find an explanation was the travel of the groups of isobars across the map. That essential part of the phenomena was ignored, being regarded as a matter that could be dealt with separately without disturbing the elements of the cyclone visible on the map. When allowance was made for the motion of the group of isobars which form a travelling cyclone the residual velocities were not those of air in rotation round the centre of isobars. We had no use for the details of the properties of fluid in permanent rotation because we could not find examples on our maps to which they could be applied.

But further inquiry showed that examples might be found in localities where they had not previously been looked for[1]. It was first realised that the isobars corresponding with a column of fluid in permanent rotation and travelling bodily across the map would not necessarily be indicated by concentric circles but might be shown by local deviations of isobars from their regular run with reference to the centre of a large cyclonic depression, such as we are accustomed to call "a small secondary." Two examples were adduced. One which was sufficiently indicated in the isobars of the maps for March 24, 1895, travelled from Cork Harbour to the mouth of the Humber at an average speed of 55 miles per hour, and then went on to the west coast of Denmark with an average speed of 82 miles per hour. Its diameter was probably about 150 miles at the beginning of the journey and 300 miles at the end. The velocities were in rough agreement with the geostrophic winds of the isobars in which the local circulation was formed. A notable peculiarity of this case was that no rain fell in the travelling cyclone. The other case was that of the tornado which visited South Wales on October 27, 1913. It was less than 10 miles in diameter and showed no disturbance of the isobars as drawn on the maps for the day. But it travelled along the line of the isobars with a velocity about three-quarters of the computed geostrophic wind. It was accompanied by very heavy rain in various localities on its route.

These examples were sufficient to show that the properties of revolving fluid are pertinent to the phenomena of travelling depressions provided that the proper centre of permanent rotation can be identified. The indication was confirmed by the occurrence of tornadoes in the southern portion of the large cyclonic depressions of the United States; the trough-line of a large

[1] Shaw, 'Revolving Fluid in the Atmosphere,' *Proc. Roy. Soc.* A, vol. XCIV, p. 33, 1917.

depression was indicated as a probable locality for their formation. At the time velocity tangential to the isobars and uniform over the area of the section of the travelling column was assumed as being a sufficient generalisation of the irregular velocities observed at the surface in different parts of a travelling cyclone. Subsequently it proved to be desirable to examine the phenomena of the travelling cyclone in relation to the properties of the normal cyclone as defined in the previous chapter and the propriety of including in a similar category other cyclonic depressions with properly chosen centres became evident. It may be noted that in doing so we approach the question of the cyclone from a standpoint which is different from that which Galton indicated. We regard the incurvature of the surface-winds not as the primary step from which all the rest of the phenomena are derived but merely as incidental to the retardation of the lowest layers of the revolving air by the friction of the ground. We make no hypothesis as to how the air at some unknown height above the ground comes to be in rotation. The question to which we address ourselves is what happens to the revolving air during its life-history as a definite travelling mass.

We shall evidently be on safe ground in applying the properties of revolving fluid provided we think of the properties as related to the centre of the rotating mass and not to the centre of instantaneous rotation.

"Such simple conclusions from the dynamics of revolving fluid as are within our reach" have been set out by Lord Rayleigh[1] for the reason that "so much of meteorology depends ultimately upon their study"; we cannot, therefore, do better than appropriate his conclusions. They deal with rotation about a fixed axis and make no allowance for the rotation of the earth. This limitation must be borne in mind in considering the distribution of pressure appropriate to the field of motion which is indicated.

The reasoning is based upon the fundamental equations of hydrodynamics, adapted to cylindrical coordinates r, θ, z with velocities u, v, w reckoned respectively in the direction of r, θ, z, increasing. For the present purposes assuming symmetry with regard to the vertical axis the equations become

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} - \frac{v^2}{r} + w\frac{\partial u}{\partial z} = -\frac{\partial P}{\partial r} \qquad \ldots\ldots(1),$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + \frac{uv}{r} + w\frac{\partial v}{\partial z} = 0 \qquad \ldots\ldots(2),$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = -\frac{\partial P}{\partial z} \qquad \ldots\ldots(3),$$

where P (assumed to be single-valued and also independent of θ) is equal to

$$\int dp/\rho - V \qquad \ldots\ldots(4).$$

V is the potential of the extraneous forces of which only the force of gravity need be considered for present purposes.

We may take in order the properties which Lord Rayleigh deduces from these equations.

[1] 'On the Dynamics of Revolving Fluid,' *Proc. Roy. Soc.* A, vol. XCIII, p. 148, 1917.

1. *Persistence of "circulation" and conservation of angular momentum.* Equation (2) may be written

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial r} + w\frac{\partial}{\partial z}\right)(rv) = 0 \qquad \text{......(5).}$$

This signifies that rv may be considered to move with the fluid. If r_0, v_0 be the initial values of r, v for any particle of the fluid, the value of v at any future time when the particle is at a distance r from the axis is given by

$$rv = r_0 v_0.$$

2. *Motion in vertical planes through the axis of* z, as for example the ascent of air near the axis and its descent in the outer region or *vice versa.*

The motion is the same as that which might occur if $v = 0$ with the addition of the centrifugal acceleration v^2/r along r.

3. *Distribution of pressure when there is no radial or vertical motion.* In this case $u = 0$ and $w = 0$, and it follows from (3) that P is independent of z and therefore a function of r and t only. From (1) it follows that v is also a function of r only and $P = \int v^2 dr/r$. Accordingly by (4)

$$\int dp/\rho = V + \int v^2 r^{-1} dr \qquad \text{......(6).}$$

V is the potential of the impressed forces; and, if gravity (acting downwards) is the only extraneous force, V is equal to $C - gz$ whence

$$\int dp/\rho = C - gz + \int v^2 dr/r.$$

For the solution of this equation we require to know the conditions of the motion in respect of the relation of p and ρ. For an incompressible fluid ρ would be constant. For isothermal variations Boyle's law ($p = a^2\rho$) would hold and Lord Rayleigh deals with no other alternative. In the free atmosphere the changes must be regarded as approximately adiabatic and the relation of p and ρ will depend upon whether the air is saturated or not. Assuming that the changes do not pass saturation we may write

$$p = (\epsilon\rho)^\gamma,$$

where ϵ depends upon the entropy of the sample of air. Hence

$$\frac{\epsilon\gamma}{\gamma - 1} p^{(\gamma-1)/\gamma} = \rho - gz + \int v^2 dr/r \qquad \text{......(7).}$$

At a constant level dp/dr is always positive. Pressure and consequently density diminish as the axis is approached. But the rarefaction near the axis does not cause the fluid there to ascend. The denser fluid outside is prevented from approaching the centre by the centrifugal force. This conclusion would be modified for the bottom layer of the atmosphere where there is loss of circulation in consequence of the eddy-motion of the air over the ground.

4. *Stability of the motion.* The equilibrium represented by equation (7) will hold good whatever may be the relation between v and r, but the motion might be unstable. The instability in the atmosphere with which we are most familiar is that of layers of air of which the heavier is above the lighter or to speak more technically the higher has less entropy or potential temperature

than those beneath. If the atmosphere is isentropic the equilibrium is neutral. So a series of rings of fluid may be in equilibrium but unstable because the outer rings are not revolving fast enough. The rotational equilibrium is neutral if vr is uniform over the area at any level because when motion takes place in one level vr remains constant and equal to k for a ring consisting always of the same matter and the centrifugal force acting upon a given portion of fluid is k^2/r^3. It is stable for fluid revolving one way between coaxial cylindrical walls only under the condition that the circulation $2\pi k$ increases with r. The conclusion is confirmed by the consideration of the change of kinetic energy on the interchange of two rings.

The result is applied to two cases of fluid moving between an inner cylinder and an outer cylinder. If the inner one rotates while the outer is fixed the equilibrium requires that the circulation should diminish outwards and therefore the motion of an inviscid fluid in that case would be unstable. On the other hand, if the outer cylinder rotates while the inner is fixed the motion satisfies the condition that the circulation increases outwards and so the motion of an inviscid fluid would be stable.

From these conclusions we infer that, regarding the motion of the margin of a column of revolving fluid forming a cyclone as the rotation of the inner cylinder, the motion caused in the air which surrounds it will be unstable, and presumably its energy will gradually be dissipated, whereas if we may regard the air-currents round the margin of an anticyclone as a rotating outer wall the motion caused in the air within will be stable.

Considerations of the stability of the motion of revolving fluid must be regarded as of vital importance in the study of the dynamics of cyclones and anticyclones in so far as they are examples of revolving fluid. The mere fact of the obvious persistence of the motion of rotation of cyclones is in itself remarkable considering that it is maintained in an environment that has no intrinsic cohesion. An outer cylinder can only be represented by the pressure of the surrounding air and an inner cylinder by the pressure of a column of revolving air. In watching weather-maps we sometimes see "secondaries" absorbed into one primary and sometimes, as in one of the cases in the *Life-History*, what was originally a secondary may absorb its own primary. These are apparently cases in which the stability of the motion round the ultimately successful centre was overpowering. What we would like to know is whether the energy of temporary disturbance of the motion due to local convection at some point away from the centre can be absorbed in the motion round the original centre and the intensity of the system grow in that way by successive slight additions in different parts of its area in the same way as a boy's whip-top can acquire speed of rotation from impulses that would meet with no such response if it were not for the stability of the motion already possessed by the top. But the air has no rigidity nor has its motion the stability which rigidity gives, and in spite of that it can acquire stability. Apparently sometimes one centre, sometimes another is favoured and the conditions of preference are unknown to us.

5. *The distribution of velocity and the variation of pressure in the outer region when there is convergence towards the axis.* This is approached by assuming that u is a function of r and t only and that $w = 0$ or at most a finite constant. Pressure may be supposed to be kept constant at the axis or preferably at an inner cylindrical boundary by the removal of fluid from within a certain radius. This is the idea of the central portion of the air of a cyclone being removed by upward convection which is in the minds of many meteorologists as the fundamental conception of a cyclone but which can be only vaguely supported by the actual phenomena of weather which accompany an ordinary cyclone.

On the hypothesis that $w = 0$ or constant the motion is two-dimensional and it may be conveniently expressed by means of the vorticity ζ, which moves with the fluid, and the stream-function ψ connected with ζ by the equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\theta^2} = 2\zeta \qquad \text{......(8).}$$

The appropriate solution is

$$\psi = 2\int r^{-1}dr\int\zeta r dr + A\log r + B\theta \qquad \text{......(9),}$$

where A and B are arbitrary constants of integration. Accordingly

$$u = -\frac{\partial\psi}{r\partial\theta} = -\frac{B}{r}, \qquad v = \frac{\partial\psi}{\partial r} = \frac{2}{r}\int\zeta r dr + \frac{A}{r} \qquad \text{......(10).}$$

In general A and B are functions of the time and ζ is a function of the time as well as of r.

If ζ is initially and therefore permanently uniform throughout the fluid

$$v = \zeta r + Ar^{-1} \qquad \text{......(11),}$$

and Lord Rayleigh remarks that this equation is still applicable under appropriate boundary conditions even when the fluid is viscous. In the case of the normal cyclone $v = \zeta r$ and therefore $A = 0$. But if the central portion of the cyclone be removed and the outer boundary closes in from R_0 initially to R at time t, since vr remains unchanged for each ring of fluid we get

$$v/\zeta = r + (R_0^2 - R^2)\, r^{-1} \qquad \text{......(12).}$$

And thus convergence towards the axis in a normal cyclone causes the fluid to acquire in addition the motion of a simple vortex of intensity increasing as R diminishes.

If at any stage the convergence ceases (6) gives $dp/dr = \rho v^2/r$ and neglecting the variations of density

$$p/\rho = \zeta^2\left\{\tfrac{1}{2}r^2 + 2\,(R_0^2 - R^2)\log r - \tfrac{1}{2}\,(R_0^2 - R^2)^2\, r^{-2}\right\} + \text{const.} \quad (13).$$

Since v^2 as a function of r continually increases as R diminishes the same is true for the difference of pressures at two given values of r, say r_1 and r_2, where r_2 is greater than r_1. Hence if by removal of air or by any other process the pressure is maintained constant at r_1, it must continually increase at r_2, or in

meteorological language convergence to a central region at which the pressure remains constant will require an increase of the gradient of the cyclone by increase of the pressure in the outer rings.

Lord Rayleigh concludes by explaining that when the u, w motion is slow relatively to the v motion we may formulate a general idea of the solution of the problem. When revolving fluid is drawn off from a point near the axis of rotation there is a tendency for the surfaces of constant circulation to retain their form and position the more pronounced the greater the speed of rotation. The escaping fluid is therefore drawn off along the axis and not symmetrically from all directions as when there is no rotation. Thus we may conclude that convection near the axis of a column of rotating fluid in the atmosphere will increase the gradient of pressure in the column provided that the air which is removed is disposed of without altering the other conditions of the environment.

In appropriating these propositions as contributing to the comprehension of the phenomena of weather we have accepted Lord Rayleigh's suggestion and must now remark that a careful analysis of the phenomena of the atmosphere is necessary in order to find examples to which the conclusions can be applied. The direct motive of their exposition was to give an analytical representation of experimental results which J. Aitken[1] had previously put forward as contributions to the study of cyclones and anticyclones. We therefore briefly recapitulate the experiments described. What concerns us chiefly in this part of the subject is the apparatus with which the experiments are conducted because in such cases the reader requires to think for himself whether the conditions and arrangements prescribed for the experiments have their counterpart in the atmosphere.

In Part I, in order to show that some initial motion of the fluid is necessary for a vortex to be formed, a vessel of water is used with a plug at the bottom that can be operated from the outside, and subsequently provision is made for the position of the opening with reference to the circumference to be changed in order to show that if there is a difference of current on two sides of the axis the motion of the vortex is with the stronger current. The same apparatus is used to show that the actual velocity of motion in the vortex is increased in a notable degree as the centre is approached, and this conclusion is further illustrated by a rotating system consisting of two balls which can be made to approach the axis of rotation while the whole system is spinning, when it is seen that the actual energy of motion of the balls increases as they get nearer together. This is in accordance with the conclusion that vr is constant.

For making experiments upon cyclonic movements in air a metal tube 15 cm. in diameter and 2 m. high was used. At the lower end of the tube was

[1] 'Notes on the Dynamics of Cyclones and Anticyclones,' by John Aitken, F.R.S., Parts I and II. *Trans. Roy. Soc. Edin.*, vol. XL, p. 131, 1901; Part III. *Proc. Roy. Soc. Edin.*, vol. XXXVI, p. 174, 1916. 'Revolving Fluid in the Atmosphere,' *Proc. Roy. Soc.*, vol. XCIV, p. 250, 1918.

a circular disc 75 cm. in diameter supported on three legs 15 cm. high, thus leaving a space of 15 cm. between the disc and the table on which it rests. To produce an up-draught jets of gas were fitted inside the tube near the lower end. To study the circulation between the disc and the table a number of light vanes, or fumes of hydrochloric acid and ammonia, were used. With this apparatus it is shown that the air moves radially towards the chimney if there is no initial movement in the air before the up-draught is started. An initial movement equally strong at all points is not of much use in generating cyclonic movement; but if the current on one side is cut off by a screen a violent cyclonic motion results in which the fumes are carried to the chimney in graceful ascending spirals. It is the lowest stratum of air that is drawn to the very centre. A further effect of the tangential motion is that the lower end of the cyclone bends away from under the centre of the apparatus, moving in the direction of the tangential current. Similar experiments can be made simply with a good fire, a free going chimney and a wet towel with a suitable arrangement of the draughts of the room. In this case when the wet towel is held vertically in front of the fire the steam is formed into a horizontal column of revolving air leading from the towel to the chimney.

In Part II the apparatus used is a large sheet of metal 75 cm. square forming a platform which can be heated by gas burners underneath, or otherwise, and from which wreaths of steam rise irregularly, when the hot surface is covered by wet cloth or paper, unless there is a definite current of air which passes over one part of the platform and misses the other. In that case the rising steam is gathered up into small cyclones which may reach a height of a metre or more above the platform and which travel across the platform with the characteristic features of the eddies that are sometimes seen in the open air.

In Part III the formation of a horizontal whirl between a wet towel and the chimney over a good fire is depicted, and further a useful modification of the arrangement for forming a vortex in water, by replacing the plug in the bottom by a siphon drawing water from the top, is described and figured and attention is called to the narrowness of the vortex produced in that way. This feature is dealt with in Lord Rayleigh's paper and it will be useful to bear it in mind when we come to consider the transmission of a circular field of pressure from the upper air to the surface.

It may be remarked that the experiments which Aitken describes are really experiments illustrating eddy-motion. Others are to be found elsewhere. In the Science Museum at South Kensington is a glass chamber with apparatus devised and constructed by W. H. Dines, in which a fine vortex of steam about a metre and a half in height can be developed at will; a gas jet below a water vessel in the bottom of the glass chamber is used to form the steam and a small exhaust fan in the ceiling provides the necessary convection.

Many experiments with eddy-motion in closed vessels are described by C. L. Weyher[1], and other authors have given descriptions of experimental

[1] C. L. Weyher, *Sur les Tourbillons*, Gauthier-Villars, 1889.

illustrations. But the question whether they are really representative of cyclonic motion in the atmosphere depends upon the precise analogy of the conditions. In the free atmosphere we miss the rigid boundaries and discontinuities which so often form an essential part of the experimental apparatus. On the other hand we have the ubiquitous effect of the rotation of the earth. There is a quasi-rigidity attaching to the motion of flexible chains under tension, but in developing its own species of rigidity with only a single coefficient of elasticity the atmosphere has to rely entirely upon the distribution of pressure for preventing general disruption. Hence we are thrown back upon the relation of pressure to wind as after all the chief consideration of the dynamics of the atmosphere.

In his paper on revolving fluid before the Royal Society, Aitken describes a new experiment of great interest designed to illustrate the effect of rain in replacing dusty air by clear air at moderate heights. A large circular flat-bottomed vessel was filled with water in which a little fine sawdust was mixed to show its movements. The water was set in circular motion with a steady flow: sand was then dropped into the water to imitate falling rain; the current was followed round with the hand so as always to drop the sand in the same part of the rotating water. "When this was done a quite well-formed eddy or cyclone was observed which travelled round in the vessel at the distance from the centre at which the sand was dropped in. It was in miniature like a secondary cyclone moving in the current of the large cyclone revolving in the vessel. Only a small quantity of sand is required to produce the result, 1 or 2 g. being sufficient." It is usual to regard the convection which precedes the rain as the cause of the secondary and the idea that the secondary may be produced by the dynamical effect of the falling rain is certainly novel and adds another to the marvellous varieties of eddy-motion.

In considering the meteorological application of experiments the fitting of the several models to the actual phenomena of the atmosphere requires the utmost caution. One has only to consider the difference between the ordinary phenomena of convection as illustrated in the laboratory and the corresponding phenomena on the large scale in the atmosphere where the conditions of the environment are of at least as much importance as the local conditions of the air which rises or falls, in order to realise that the scale of the free atmosphere introduces conditions peculiar to itself.

With regard to the application of Lord Rayleigh's equations and Aitken's experimental illustrations to the phenomena of cyclonic depressions or anticyclones as we find them represented by meteorological observations we ought to devote our attention especially to the details of the motion of the air and of the distribution of pressure and density within the cyclone or anticyclone because those are the elements which enter into the equations and are represented in the experiments. This object is by no means easy of attainment with the material at our disposal because there is not enough information to enable us to complete the picture with the accuracy that a rigorous comparison requires. We have already cited two instances in which the idea of rotating

columns of air appears justified but they are on a comparatively small scale and the details of distribution within the areas identified are too meagre for a proper comparison. In the cases of the fast travelling storms of September 10–11, 1903, and March 24–25, 1902, we have sufficient detail to show that the phenomena are fairly well represented by a rotating column of fluid with an appropriate centre and we shall presently give some noteworthy evidence as to the distribution of velocity with reference to the centre of the September storm at one point of its course. In considering cyclones of larger diameter we are faced with the difficulty that convection which may be represented by rainfall alters, for the time being at least, the regularity of the distribution of pressure and wind. We have no satisfactory expression for the effect of local convection outside the central region. We may surmise that it will cause a local circulation and if sufficiently prolonged may give rise to local rotation of sufficient extent and intensity to become recognised as a secondary depression and ultimately, if conditions are favourable, may absorb the circulation of the original system.

We have, however, some guidance from Lord Rayleigh's equations as to the effect of convection near the core of the revolving column and it will be useful to add some further considerations as to the process, regarding separately in accordance with proposition (5) that part of it which is operative during the convection and that part of it which follows when the convection has ceased and the rotation continues.

While the convection is operative within the core of a revolving column in the upper air it has been shown that air will be drawn from the core of the column below, not from the cylindrical walls which surround the rising air; the question of the statical equilibrium of the core of the column beneath the original locus of convection does not enter immediately into the solution.

Convection will take place spontaneously when the successive layers of air are so arranged that the entropy or potential temperature of the lower strata is greater than that of the higher and will go on until that state of affairs is ended. How the ascended air distributes itself we cannot say because we have no adequate means of forming an opinion. The shape of an anvil cloud forming the top of a cumulo-nimbus cloud may give us an indication on a comparatively small scale. We have assumed that the motion of clouds in the upper levels gives us an indication of the distribution of pressure at those levels. We cannot therefore regard it as independent of that distribution and we cannot coordinate their motion with the ascent of air from down below until we know what the distribution of pressure in the upper air is.

Our knowledge of the phenomena at the top of a revolving column of air when convection is active at the core is therefore very defective, we can only say vaguely that the convection will go on until the conditions for thermodynamic stability are satisfied. The ascending air cannot as a rule pass a layer of inversion of lapse of temperature nor penetrate far into an isothermal region, a region of no lapse. When the thermodynamic conditions are satisfied the rotation of the air prevents the flow of air from the sides to the core and

the thermal conditions prevent the air filling up the low pressure from the top unless the column travels from under the air which has risen to other air of lower entropy. There will be a certain amount of flow inwards at the bottom but that will be restricted by the isentropic conditions of the ascent and some considerable time will be required to disintegrate the rotation. If, therefore, the column be finished off at the top in such a way as to prevent the low-pressure filling up there the revolving column might travel for a considerable period without material change. Protection of that kind might be afforded if the column were surmounted by a cap which travelled along with the column and in which the rotation gradually diminished with height.

We have no satisfactory information as to the level at which the development of a column of rotating fluid begins nor how far up it extends. If it originates spontaneously with convection the rotation must presumably, from the experimental analogy, begin in the layer from which the ascending air is drawn, and whether it subsequently extends upwards we cannot say, but we may show that the rotating motion will always extend downwards to the earth's surface if the convection persists long enough.

The rotation will develop a circular distribution of pressure which on the principle of the transmission of fluid pressure will be transmitted to all the layers beneath. Those layers at the first setting up of the circular isobars within them will not have the rotation which keeps the fluid from moving towards the axis to fill up the low pressure; consequently the air will move inward towards the core and the part immediately below the core of the whirl will pass upward into the core; the air moving inwards in the layer beneath will gradually develop rotation which will balance the distribution of pressure, and so the core will gradually be drawn out of the column beneath and rotation set up provided that the convection up above is strong enough to carry with it the air supplied from the core of the column beneath. Thus the sudden creation of a circular field of pressure due to the convection will set up a sort of trunk of "suction" along the core which will extend further downwards as the rotation gradually develops and ultimately reach the ground. If the original instability is very marked it seems possible that the suction at the ground, when the core reaches it, might be very strong and the inrush and uprush of air near it very powerful. This process on a large scale might account for the carrying up of dust, sand, small fish and other objects into the core and so upwards into the air[1].

And here it is important to notice in continuation of the proposition, explained in chapter X, as to the superposition of a uniform linear field of pressure upon a circular field, that the transmission to the surface of a circular field of pressure suddenly created by convection or otherwise in the upper air would form a circular field at the surface within a field of straight isobars, but the centre would not be vertically underneath the centre of the circular field in the upper air. There would be displacement of the centre of circular isobars through the distance $V/(2\omega \sin \phi + \zeta)$ where ζ is the angular velocity of the

[1] Cf. *Nature*, vol. CII, p. 46. *Q. J. Roy. Met. Soc.*, vol. XLIV, p. 270, 1918.

original rotation and V is equal to $\beta/(\rho\zeta)$, β being the gradient of the superposed field, in this case the gradient of the field due to the layer between the original level of the whirl and the ground. The added gradient of this layer will be represented by the change in the flow of momentum in the layer. Hence we may conclude that the core of a column descending to the surface will not be along a vertical line but to a point on the surface displaced from the vertical across the wind of the lower layers. The air will be drawn out along the line of the core whether it be vertical or sloped at the angle defined by the superposed gradient. In this we may find an explanation of the resemblance of water-spouts to elephants' trunks, and we may also conclude in general that the core of a column of revolving fluid will not be vertical and that the position of the core at any level will depend upon the gradient, at that level, of the isobars within which the rotation takes place. It has long been surmised that the axis of a cyclone is inclined to the vertical and the considerations here set out add definiteness to the meaning of that idea. Opportunities for definite evidence upon the subject are rare but we may cite an instance of a pilot-balloon ascent close to the core of a cyclone at 17 h. 15 m. on 28th March, 1918.

The point of observation, near Leith, was about 100 kilometres due North of the centre of a well-marked circular depression which was complete up to 800 kilometres in diameter.

The observations were:

Height	1	2	3	4	5	6	7	8	9	10	k
Wind-velocity	4	3	3	3	4	6	9	12	9	8	m/s
Wind-direction	E	SE	S	SE	S	SW	S	S	S	S	

Thus the results give a South wind above the Easterly wind on the surface and point therefore to the displacement of the core towards the North West in the upper air.

From these preliminary considerations we may pass to Lord Rayleigh's equation (12) as representing the distribution of velocity in a cyclone originally normal in which convection at the core, and consequently convergence, is operative, and equation (13) as representing the distribution of pressure at the stage when the convergence ceases. Since the distribution of pressure in a cyclone of considerable dimensions will alter very slowly, we can regard the pressure equation as holding good for the specified distribution of velocity although the convergence may not have ceased. And the two equations may be regarded as general equations for the velocity and pressure in a cyclone originally normal but affected by convection at the core or any other process which is equivalent thereto and causes the convergence towards the centre of the circles of fluid in any layer.

For the purpose of rigorous comparison we ought to make allowance in the equations for the rotation of the earth. For this purpose as a first approximation, neglecting the earth's curvature, we should increase the radial acceleration v^2/r in equation (1) by $2\omega v \sin\phi$, and subsequent equations depending upon it would be modified in consequence. And if the cyclone is travelling

as our cyclones travel we should allow for the effect of the motion of translation. That may perhaps be sufficiently provided for by understanding that the velocity should represent the velocity relative to the tornado centre and not the resultant velocity of the air and bearing in mind the displacement of the centre of circular isobars with reference to the centres of permanent and instantaneous rotation. We cannot assume even for the purposes of rough approximation, that the effect of the geostrophic term which ought to appear in equation (1) may be ignored in our consideration of the use of the two equations to represent the phenomena of the cyclones of our experience.

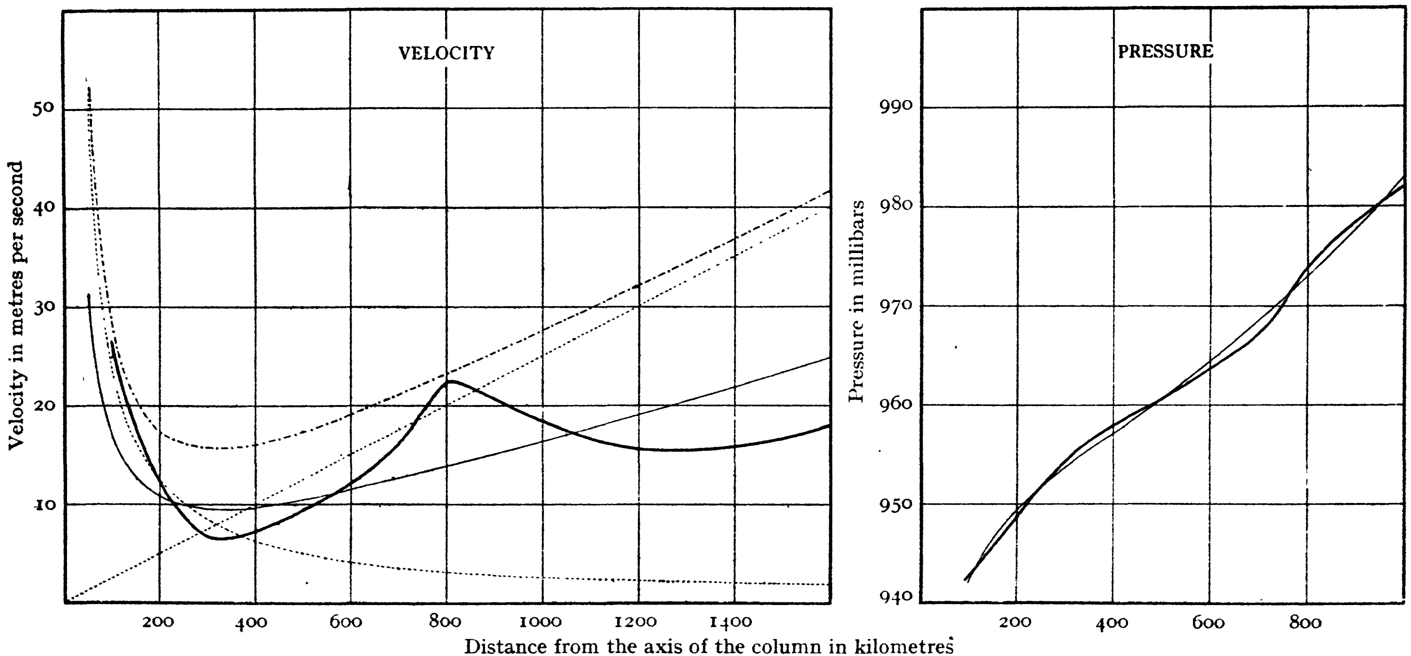
We can give a general idea of the distribution of velocity and pressure in a cyclone that has experienced convergence towards the centre by presenting graphs on an arbitrary scale of the several terms of the right-hand side of equation (12) and their resultant and the graph for the resultant distribution of pressure as given by equation (13), modified by the addition of $2\omega v \sin \phi$ to the pressure equation from which equation (13) is derived by direct integration. These graphs are all included in fig. 1.

To provide material for comparison with actual cyclonic conditions we can appeal in the first instance to maps because they present the actual position at a specific epoch. We select the most conspicuous example known to us of a well-formed cyclone of the largest scale which is that of February 20, 1907, memorable for the gale which wrecked the s.s. *Berlin* off the Hook of Holland. We can regard it as a huge cap of air instantaneously in rotation about a centre very close to the West coast of Norway in latitude 60° N. in the North Atlantic not far from the centre of isobars. Taking the figures for pressure and velocity from the observations at exposed stations in the south-western sector of the cyclone from the map for 8 h. of the day of the storm[1] we get the curves represented by the thick lines of fig. 1. The resultant graphs obtained from the equations are represented by continuous thin lines: the auxiliary graphs by dotted lines. The similarity of the actual to the theoretical curves is sufficiently well marked to justify the comparison. It must be remembered that the winds are surface-winds and the velocities of the theoretical wind of the free atmosphere have been reduced by subtracting a third for the comparison. We ought also to note that there are in the map some local peculiarities which are practically of the highest importance because among them was the line-squall near the trough-line which caused the wreck referred to. Such local disturbances are not represented in our diagrams, because the south-western sector was generally free from them.

Another mode of comparison with the theoretical results may be based on the autographic records of pressure and wind obtained at the various observatories within the area covered by a depression. This method has the great advantage attaching to continuous records, but it is not quite so appropriate as a perfectly complete map would be because the record which we obtain depends upon the travel of the disturbance over the station and changes in the distribution of pressure and velocity certainly may take place while the

[1] *Weather of the British Coasts*, chap. XI, § 7.

FIG. 1. Graphs of the distribution of velocity and pressure in the base of a column of revolving fluid, in latitude 60°, according to the formulae $v = \zeta\,(r + Ar^{-1})$ and $dp/dr = \rho v^2 r^{-1} + 2\omega v \sin\phi$, where $\zeta = 2{\cdot}5 \times 10^{-5}$ radians per second and $\zeta A = 2{\cdot}5 \times 10^{6}$ C.G.S. units, compared with the distribution of velocity and pressure in the SW quadrant of the storm of February 20, 1907, at 8 h.



The dotted lines .... represent the components of velocity depending upon ζr and $\zeta A r^{-1}$ respectively, and the chain line ---·---·--- the combination of the two. The thin full lines represent the computed velocity (reduced in the ratio of 6:10 to allow for friction at the surface) and the computed pressure, respectively. The thick lines represent the velocity and pressure as recorded on the map on February 20, 1907 (*Weather of the British Coasts*, p. 122), in the SW quadrant of the storm.

record is in progress and probably more or less material changes occur in every case.

For the purpose of comparison we may use records either of wind or of pressure. Records of temperature are of little or no use for the purpose of identifying the structure of a cyclonic depression because the temperature of layers near the surface depends to so great an extent on the recent history of the air and at the surface is subject to local variations from which the upper air is probably free. Humidity and sunshine are similarly dependent upon local circumstances.

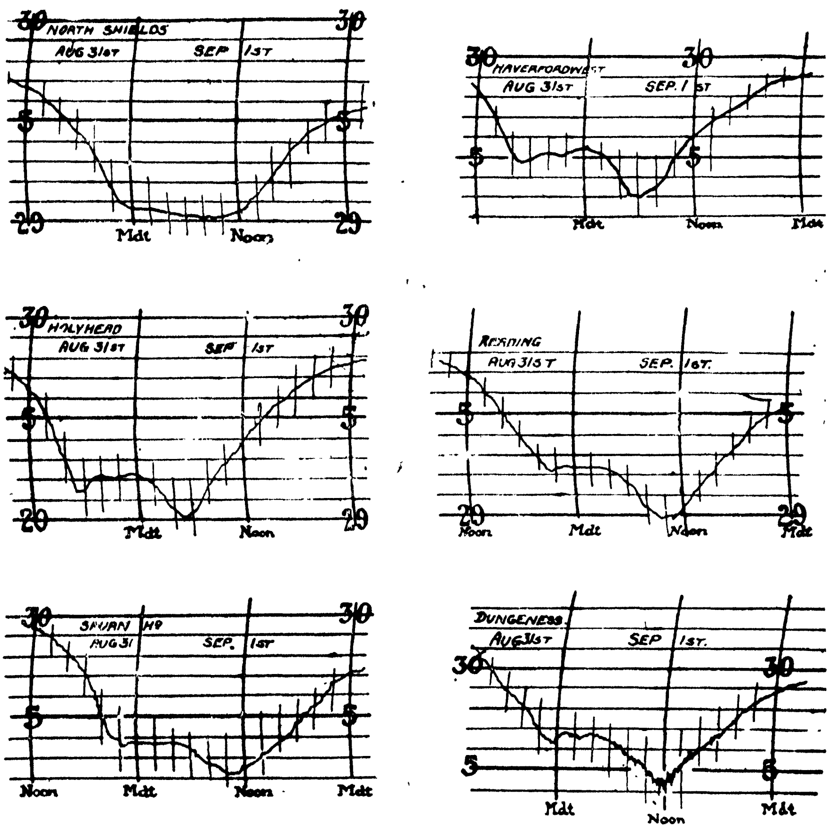


FIG. 2. Graphs of pressure showing interference of the surface layers with the symmetry of revolving fluid.

Records of pressure are familiar to meteorologists but the analysis by inspection of their evidence in illustration of revolving fluid in the atmosphere is not an easy matter. We may first call attention to a peculiarity that is noticeable in many of them in connexion with the passage of the trough, and may be attributed to the effect of the juxtaposition of bodies of air of different temperatures in the lower layers which destroys the symmetry of the distribution of pressure with regard to the centre. The motion of these heterogeneous masses of air is controlled by the distribution of pressure above them

and in turn affects the distribution at the surface. The inherent difficulty about surface-temperature in respect of the dynamics of the atmosphere depends upon its unilateral behaviour. In the free air all changes of temperature can find their adjustment in a mutually reciprocal manner. Cold air may go downward, warm air upward, but at the surface the air which is cooled must remain and in consequence the lowest layers of the air beneath a revolving column may be divided into portions of which the temperature is markedly different. Where these different bodies of air are in juxtaposition line-squalls occur and the symmetry of the curve representing the variation of the barometer is destroyed. In illustration of this interference with the symmetry characteristic of revolving fluid we may show the differences noticeable in six barograms representing the same cyclonic depression (fig. 2). On this account, ordinary barographic records do not generally help us in our study of the details of the relation of the phenomena of revolving fluid to those of cyclonic depressions in our latitudes. On the other hand the available examples of barograms of tropical revolving storms show the appropriate symmetry. An example is represented in the *Meteorological Glossary*[1]. Occasionally however at different stations we have obtained curves representing variations of pressure which may be attributed to revolving fluid. As an example we may take the simultaneous records from four micro-barographs which magnify the pressure-variation twenty fold represented reduced to half-size in fig. 3. They are compiled from the records of July 12, 1908, which are reproduced in the Report of the British Association for that year[2].

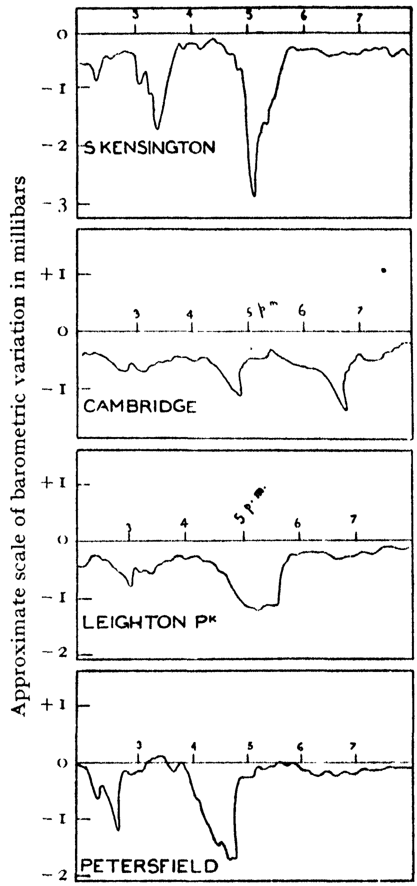


FIG. 3. Micro-barograms.

Fluctuations such as are indicated in this figure occur in what may be called thundery weather but at some hundred miles away from the locality where the thunderstorm is taking place. The largest of those represented in the figure was accompanied by heavy clouds and a few drops of rain but nothing more. It seems hardly possible to regard them otherwise than as local whirls

[1] M. O. Publication, 225 ii, s.v. Hurricanes.
[2] *British Association Report*, 1908, p. 608.

of air, though we have no actual evidence of wind-velocity in favour of that view, because there seems no explanation of the maintenance of the pressure-distribution for the duration indicated in the diagram on any other hypothesis.

As examples of records of wind we have the four curves of fig. 4 which give smoothed curves of variation of wind during the passage of depressions on four occasions for which facsimiles of the original records appear in the First Report of the Advisory Committee for Aeronautics[1]. The curves have been obtained by drawing lines along the middle of the ribbon of the original trace.

The first, A, represents the changes in the velocity of the wind as a depression passed Aberdeen on the North. It is characteristic of records of such occurrences. The variation of direction was from SW by S through W to NW, so that the actual centre missed the Observatory; and it should be noted that the velocity increases to a maximum when the wind is from WNW and the centre is nearest. From that point if falls off again. The shape of the curve suggests therefore the simple vortex with velocity proportional to $1/r$ rather than the normal cyclone with velocity proportional to r.

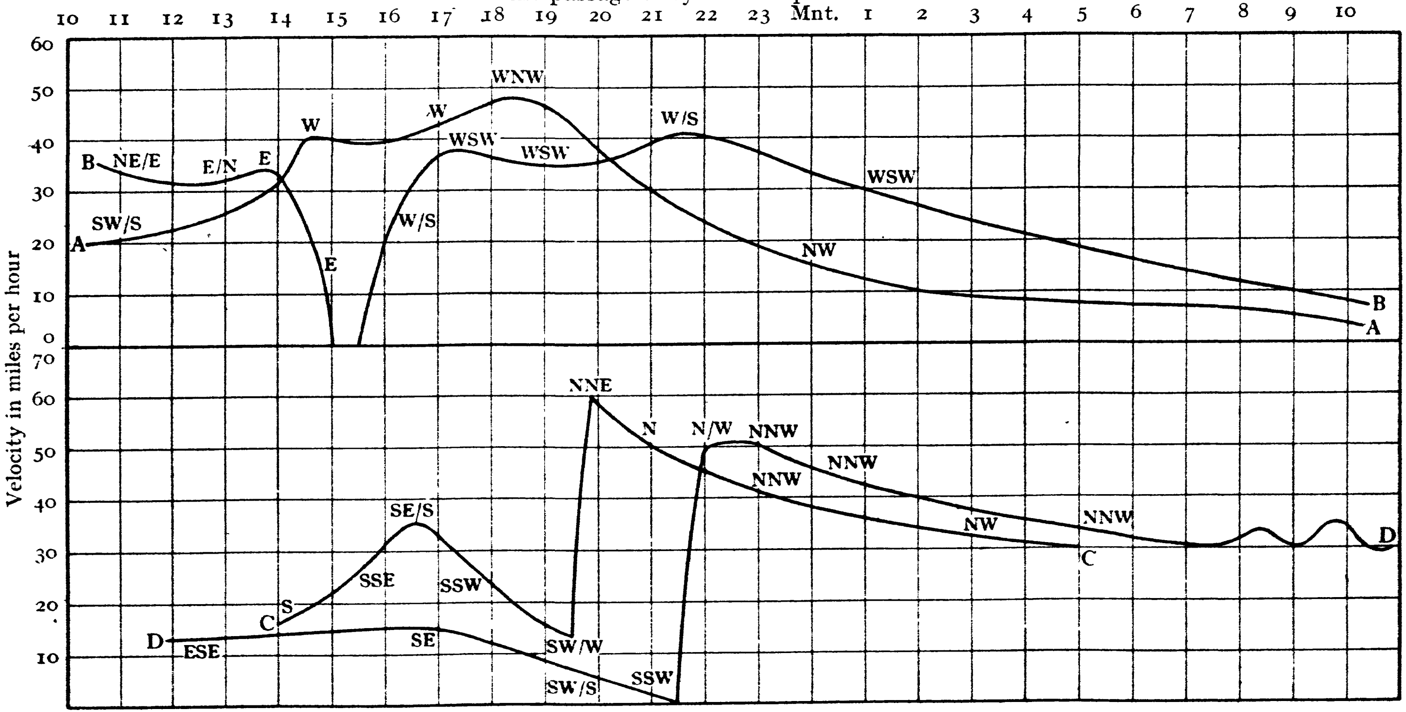
The next example, B, is from Scilly for January 9–10, 1901, which shows a sudden lull and recovery in the wind. The shape of the curve suggests a normal cyclone with great angular velocity comprised within a space equivalent to an hour-and-a-half on each side of the centre which passed from South to North: beyond those limits the velocity is nearly uniform for a long time before and after the passage of the centre but in opposite directions on either side of it.

The third example, C, shows remarkable changes in wind at Holyhead during the passage of the storm of September 10–11, 1903, which has already been represented in fig. 4 of chap. X and which will be within the recollection of all those who were present at the meeting of the British Association at Southport in that year. The sudden change of direction from SW by W to NNE, N and NW shows that there was a centre to the North of the anemometer which passed from West to East but there is no proper symmetry with regard to the centre. At first we supposed that an explanation of this want of symmetry might be found by assuming that there were two whirls both of the order A/r superposed and not quite concentric, and that the smaller one escaped notice in the map of the depression in chap. X.

And this may remind us that there are endless possibilities for one column of revolving fluid within another because, as we have seen in chap. X, a revolving column can be carried along with the flow along the isobars. Hence if the primary analysis of the distribution of winds over the Northern hemisphere be a rotation from West to East round the pole covering in the more Northern regions a rotation from East to West represented by a great anticyclone, then within either the Westerly or the Easterly current or between them there may be examples of revolving fluid of which the whirl of February 20, 1907, is a most notable specimen, and within the area of a cyclonic

[1] *Reports and Memoranda*, No. 9, Details of Wind Structure. Figures 11*a*, 16, 9*d* and 17.

FIG. 4. Smoothed middle lines of records of tube-anemographs which are reproduced in facsimile in the report on Wind Structure (First Report of the Advisory Committee for Aeronautics, 1910), illustrating typical distributions of velocity during the passage of cyclonic depressions.



A. Aberdeen, 1908; January 6–7. The centre of a depression passed in the North.
B. Scilly, 1901, January 9–10. The centre of a depression passed directly over the anemometer from South to North.
C. Holyhead, 1903, September 10–11. The depression represented in figure 6 of chapter x passed from West to East with the centre of winds close to the anemometer.
D. Holyhead, 1900, August 3. Representing a case of discontinuity of horizontal motion.

depression of that type there may be travelling columns of revolving fluid which are identified on the map as small secondaries. Even in those we may have still smaller whirls which at most are represented by embroidery of the barogram.

But the real explanation in this case proved to be quite different. The orientation of the winds on the two sides of the centre of the depression is nearly opposite and it was realised that, as the record of wind is obtained from an anemometer only 32 feet from the ground, the record will be subject to the peculiarities of the exposure; and that, according to the figures given in Table II and the curve of relation of surface-wind to the gradient in fig. 3 of chap. II, the effect of the exposure at Holyhead is very different for different orientations. It seemed desirable therefore to "correct" the readings taken from the record in order to obtain the "free" wind for the several orientations by the application, as a multiplier, of the reciprocal of the corresponding ratio of the surface-wind to the geostrophic wind. The record, C, was therefore treated in this manner and a graph of the free wind of the depression thus obtained using the figures of Table II which were derived from an analysis of eight years of observation. The "corrected" graph is represented in fig. 6 and it will be apparent that the original want of symmetry has disappeared and we obtain a very striking curve representing the distribution of velocity in the depression of September 10, 1903, which consists of an inner portion or normal cyclone with a large angular velocity, like that in the central portion of the graph B, and an outer portion in which the variation of velocity with distance from the centre is more nearly according to the law Ar^{-1}. The depression was travelling at the time at about 44 miles per hour or 20 metres per second; and the diameter of the central portion which took three hours to pass the instrument may be estimated at about 250 kilometres. On reference to the map on p. 128 we find that the centre of isobars was a considerable distance south of Holyhead and we therefore have further direct evidence of the separation of the dynamic centre from the instantaneous centre in the case chosen as an example in chap. x. The diameter of the complete revolving system is about 1000 kilometres. The nearly uniform velocity on either side of it in the record represents the velocity of translation corresponding with the isobars of the larger system in which the rotating column moved. They were from West at first but subsequently from North West. Record C has therefore furnished us with another type of depression different in its distribution of velocity from that of February 20, 1907.

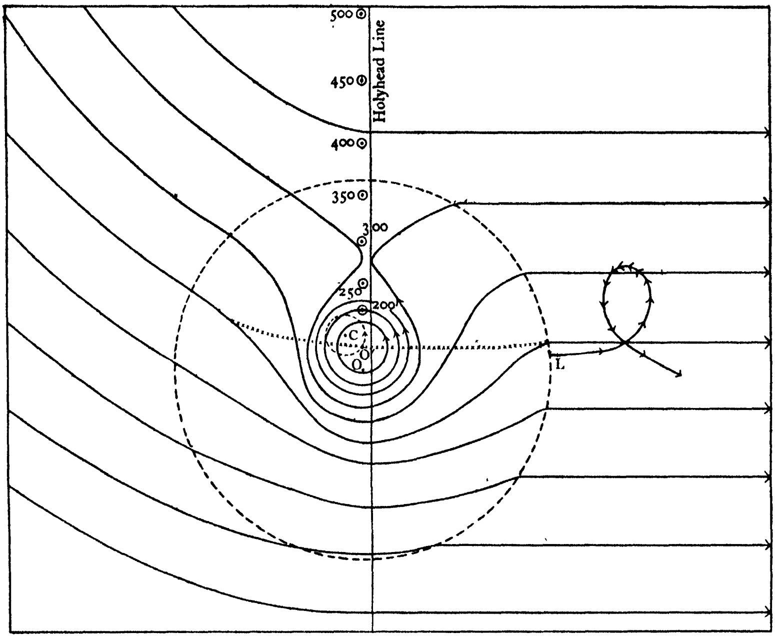
The result is interesting in another way. The velocity at its lowest does not reach the zero line as it would do at the actual centre of instantaneous rotation of a travelling column. Making a correction for the deviation by friction, the minimum velocity is from 260° (W by S) and previously from 221° (SW). Hence we may conclude that the instantaneous centre at the time of the minimum velocity was a little to the North of the anemometer at Holyhead though the centre of the circle of maximum velocity was just to the South of that station.

It is only fair to say that in Table II of chap. II there is another line of values for the ratio of the surface-wind to the geostrophic wind at Holyhead which gives a much higher factor of correction for winds in south-easterly quarters. The result of using these corrections instead of those which were employed for fig. 6 is that the SSW wind when corrected reaches the very high figure of 130 miles an hour while the NNE wind on the other side only reaches 100 miles an hour. The want of symmetry instead of disappearing is reversed. The discrepancy is to be regretted but, as we have already said, its existence is not surprising because the determination of the ratio of the surface-wind to the geostrophic wind depends upon a very rough-and-ready mode of procedure. Certainly when the figures were taken out it was never supposed that they would some day be used to correct individual readings of an anemometer of the first class; and yet, the conclusion that there was a symmetrical circulation round an instantaneous centre fits in so well with all the rest of the known circumstances of the occasion that it claims recognition in spite of the crudeness of the observations by which it has been reached.

Let us therefore pursue the study of this occasion a little further. We have already seen in chap. X that a normal cyclone with vorticity ζ travelling with velocity V is represented by instantaneous rotation round a kinematic centre distant V/ζ from the tornado centre. With a rotating system in which the velocity depends upon the distance from the centre but is not directly proportional to it the result is not so simple, but we can obtain a solution graphically. If we consider a "simple vortex" in which the velocity is proportional to $1/r$, a ring with radius r will have a vorticity proportional to $1/r^2$, and when the system travels with velocity V the particles in that particular ring will have the velocity which corresponds with rotation round a centre at a distance proportional to Vr^2. By finding the centres for a series of rings we can obtain enough indications of the velocity at different points of the resultant field to enable us to draw lines of flow for the combination of the motion of translation and the motion of rotation in the "simple vortex."

Suppose, therefore, that we regard the conditions of the travelling cyclone of September 10–11, 1903, as represented by a normal cyclone with a radius of 125 kilometres and marginal velocity of 43 metres per second occupying the central portion and surrounded by a simple vortex in which vr is constant and equal to the value indicated in the marginal ring of the normal cyclone, where v is 43 metres per second and r 125 kilometres. The product in C.G.S. units is $5{\cdot}4 \times 10^{10}$. Combining the motion of this rotating system with the translation of 20 metres per second we get the results which are represented by the lines of flow charted in fig. 5 on the scale of the larger map of the Daily Weather Report. There is purposely a little want of symmetry in the lines, introduced by the change in the direction of the stream in which the rotating system was drifting. The centres which have been used for constructing the elements out of which the lines of flow are made are indicated on the chart; the figures against the centres give the radii of the original rings.

FIG. 5. Chart for 18 h (on the scale of the 7 h chart of the Daily Weather Report, $1:2 \times 10^7$) of the lines of flow of air in a stream running at 20 metres per second, carrying with it a normal cyclone which has a diameter of 250 kilometres and vorticity ζ equal to $3{\cdot}4 \times 10^{-4}$ radians per second, and is surrounded by a "simple vortex" in the labile condition as regards stability having the product vr constant and equal to $5{\cdot}4 \times 10^{10}$ C.G.S. units. (43 metres per second at 125 kilometres.)



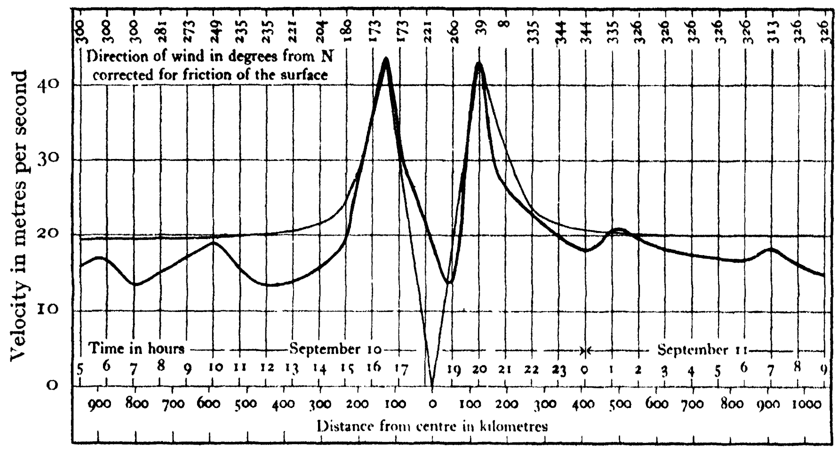
Note. In figure 5 the circle marked by a chain line has been taken to mark a column of revolving fluid which had a velocity in rotation of one metre per second. O is the tornado centre. O′ the instantaneous centre of the normal cyclone of 125 kilometres radius. C the centre of a small circle representing the actual circulation within the normal cyclone as recorded by the anemometer at Holyhead on September 10. The instantaneous centre of rotation for the rings of the simple vortex of different diameters is indicated by points against which the corresponding radius is marked in kilometres. The section of the vortex which passed over the anemometer is indicated by a line of crosses.

Then taking a line across the chart indicating as nearly as can be ascertained the section of the system which passed over the anemometer at Holyhead we obtain the velocity for successive stages as the velocity in rotation round the centre proper for the moment with the proper vorticity and radius. By these means a graph in a thin line has been added to the diagram of fig. 6 representing the velocity at different stages of the passage of the hypothetical system over the anemometer. We have already mentioned that the thick line represents the record of the Holyhead anemometer corrected for the friction at the surface. The reader must remember that the anemometer curve is read from left to right so that the later stages of the proceedings are on the right; but the chart represents what passes over the anemometer from left to right and hence the earlier stages are represented on the right and the two representations must be compared by reading in opposite ways.

As an analysis of meteorological conditions the agreement disclosed by the comparison is remarkable. No serious misrepresentation would result if

FIG. 6. Graph (in thin line) of the computed record of the wind during the passage of the hypothetical cyclonic system represented in fig. 5, together with a graph (in thick line) of the record of the wind at Holyhead from September 10, 5 h, to September 11, 9 h, 1903, corrected for the influence of the friction of the surface by the data of Table II, chap. II, to give the velocity of the wind in the "free air."

The time-scale is adjusted so that (if the graph were reversed) the velocity at any point of it would correspond with that of a point in fig. 5 at the same distance from the centre on the line of crosses which marks the section of the system represented on the anemogram for Holyhead.



Note. By an oversight in the working out of the details (at the time when the war was nearing the dramatic dénouement of November 1918) the larger dotted circle of figure 5 was taken as marking the positions in the vortex of a velocity of one metre per second instead of ten metres per second. The thin line (theoretical) of fig. 6, therefore, does not properly represent a cross section of fig. 5 but an imaginary and unstable vortex of more limited dimensions in which the velocity falls off more rapidly than in the simple vortex with *vr* constant.

The form of the theoretical curve can, however, be adjusted to a simple vortex approximating to the curve of observed velocity by selecting values for the velocity of translation and the maximum velocity in rotation. Adhering to the value of 43 m/s for the maximum velocity a very good approximation is obtained with a velocity of translation of 16 m/s and this agrees with the average rate of travel of the centre between 16 h and 24 h and also with the velocity of 17 m/s computed from the parallel isobars on the eastern side of the depression as shown in the map (fig. 4, chap. x). The value 20 m/s used in fig. 6 was taken from the positions of the centre at 18 h and 20 h on either side of Holyhead as shown in fig. 5, chap. x, and is probably too large. An alteration of the velocity of translation implies a corresponding change in the scale of distances in fig. 6 and consequently in the diagrams and in many of the numerical values quoted in the text. The lines of the argument as to the representation of a travelling cyclone by the combination of a normal cyclone, surrounded by a simple vortex, with a velocity of translation represented by straight isobars are not affected but they await confirmation by other examples. Meanwhile it has been thought best to allow the diagrams and text to stand as printed, leaving the reader who is interested in the details to work out the corrections for himself.

we regarded the cyclonic depression of September 10–11 as consisting of a central core of normal cyclone surrounded by a simple vortex, with *vr* equal to $5{\cdot}4 \times 10^{10}$ C.G.S. units.

It will be remembered that for stability *vr* ought to increase with distance and the actual example shows *vr* to be apparently just beyond the limit of stability. This is a very interesting circumstance in view of the fact that some day we may learn what the consequence of instability must be in such a case. The normal cyclone is a very stable form of motion and the wavy form of the

graph of velocity is very suggestive of successive rings of stable motion with intermediate rings of less stability. It would of course be absurd to lay great stress upon the details of the shape of a graph arrived at by the method described, but the shape and its symmetry with regard to the centre are too attractive to be passed without remark.

Perhaps we may regard the fast travelling cyclones, at least, as resembling more or less clearly the structure indicated by a simple vortex surrounding a normal cyclone, and gradually degenerating for lack of stability. That hypothesis explains another feature of the cyclone of September 10–11 which has for a long time seemed rather mysterious; that is the looped trajectory marked L on the chart of trajectories, fig. 5 of chap. x. It appears that, with the system described, there must be two points of zero motion, one within the region of the normal cyclone and the other somewhere in the simple vortex where the velocity is equal and opposite to the velocity of translation. This point is clearly indicated on the chart of fig. 5 and the air which has a westward motion just below it must form part of a trajectory such as that which is drawn on the chart from the starting point marked L. Looped trajectories are generally formed round the kinematic centre of a cyclone but they can also be formed in the peculiar circumstances indicated in fig. 5. A few examples have been found by R. Corless in an unpublished discussion of the complex cyclone of October 23, 1909.

The fourth trace, D of fig. 4, was merely introduced as an example of the discontinuity of velocity in the atmosphere as disclosed by the record of an anemometer, but after what has been said about trace C and in view of the general similitude of the two curves it seems possible that the discontinuity was more apparent than real. It should be noted that the time-scale for the trace differs from that of the other traces by twelve hours. The sudden increase of velocity occurred at 10 h on August 3, 1900. A better example of discontinuity in velocity is shown in the record at Pendennis Castle for August 11, 1903, which is reproduced in the *Life-History*. It would clearly be desirable to examine the effect of treating the other records of fig. 4 in the same way as those for September 10. A cursory inspection shows that the irregularity of the two maxima of curve A for Aberdeen would also disappear.

To the distribution of velocity represented in fig. 5 may be fitted a distribution of pressure because, as we have seen in chap. x, the field of pressure appropriate for a travelling rotating system is obtained by the combination of the field appropriate to the rotation with that appropriate to the translation. In the case of the normal cyclone it has been shown that the combination is a system of circular isobars identical with that appropriate to the rotational component but centred at a new point. The same mode of procedure enables us to deal with the central region of the scheme of fig. 5, within 125 kilometres of the centre. But in the region of the outer "vortex," where each ring of air has a different angular velocity, a whole series of new centres will be required, not merely a single one; and the equation to the family of curves representing

the isobars becomes complicated. The resultant distribution of pressure can, however, be obtained graphically. The component field of pressure in the region of the vortex appropriate to the rotation can be obtained by the integration of the gradient equation (γ) in the form

$$dp/dr = \rho v\,(2\omega \sin\phi + v/r)$$

by substituting A/r for the velocity in rotation, v, where A, in C.G.S. units, is $5{\cdot}4 \times 10^{10}$. The other component field, appropriate to the translation, is given by the gradient equation (G)

$$dp'/dy = -\,2\omega\rho V \sin\phi,$$

where for V is substituted the velocity of translation, which in C.G.S. units is 2×10^3. Plotting these two component fields upon squared paper and combining them we obtain a chart of isobars representing the resultant distribution of pressure which is quite similar in its general features to the chart of velocity as represented in fig. 5. But there are differences in detail which are specially noteworthy. There are two points of zero gradient where the gradient for the motion of rotation is equal and opposite to that for the motion of translation, just as there are two points of no velocity; but the points of zero gradient do not coincide with the points of no velocity. In the "normal cyclone" the point of zero gradient will be nearer to the centre than the point of no velocity; and in the "vortex" it will be further away. The separation of the two points of zero gradient in the resultant field of pressure will therefore be greater than the separation of the two points of no velocity, and the isobars must be adjusted to represent the situation. This state of things arises from the fact that the equation (γ) for the gradient of pressure in the field for rotation includes a term v^2/r which does not appear in the geostrophic equation (G) for the field of translation. In other words, motion along a curved path balances a greater gradient than motion with equal velocity along a straight path.

The positions of the points of zero gradient will be on the y-axis as selected for the representation of the translation, and can be computed by writing y for r in the gradient equation (γ) and equating the right-hand sides of the two equations. It is evident that the two gradients are equal and opposite when the velocity of rotation v is less than the velocity of translation V. Thus, where the gradients are equal and opposite there will be a residual velocity in the direction of translation and, on the other hand, where the velocities are equal and opposite there will be a residual gradient of pressure in favour of the rotational component. Since the smaller velocity is to be found nearer the centre in the normal cyclone and further from the centre in the simple vortex the points of zero gradient will be nearer to the centre in the one case, and further from the centre in the other case, than the two points of zero velocity respectively. The isobars which represent the distribution must be elongated to provide for the increased separation.

The equation for the determination of the point of zero gradient in the region of the vortex is a cubic which has a real root giving a distance of approximately 370 kilometres from the centre of the revolving fluid. This

solution agrees well, as of course it should do, with a chart constructed to represent the distribution of pressure. The displacement of the centre of isobars for the interior normal cyclone, for which ζ is equal to $3{\cdot}4 \times 10^{-4}$ radians per second, in a current with a flow of 2×10^3 cm/s, as computed by the formula of p. 125, is 15 kilometres. The calculated distance between the two points of zero gradient is therefore 355 kilometres. The separation of the two points of zero velocity shown in fig. 5 is 250 kilometres. Hence we see that the point of zero gradient in the region of the vortex may be a considerable distance from the point of zero velocity.

The conclusion is very well illustrated by the actual map of the distribution of pressure on the occasion of the cyclone of September 10–11, 1903, which is given in fig. 4 of chap. x. We have there the two points of zero gradient indicated, one very definitely by the small circle near to Holyhead and the other very vaguely by the bending of the isobars in the neighbourhood of Aberdeen. Making an estimate of the position of the second, the distance apart measures about 500 kilometres which is again considerably greater than that obtained from calculation, but in this connexion we ought to bear in mind the recrudescence of velocity in the actual cyclone of September 10–11, 1903, shown in the record of velocity represented in fig. 6, at 600 kilometres from the centre on the eastern side and at 500 kilometres on the western side. We have already explained that too much stress must not be laid upon these irregularities in the curve of wind-velocity, considering the process by which they were arrived at; yet it is curious, if nothing more, that a ring of enhanced velocity at 500 kilometres distance, with the pressure-gradient which would accompany it, would be extraordinarily useful in bringing the theoretical map into accord with the observed phenomena.

And now, having obtained, in a very unexpected manner, further insight into what the structure of an actual travelling cyclone really is we may profitably turn back to the representation of the theoretical normal cyclone as set out in fig. 6 of chap. x. In describing the map we explained that the theoretical cyclone therein represented ended in a ring of maximum velocity which had to be adjusted to its environment and we had no information as to what the nature of the adjustment was. We left it as a discontinuity of velocity which had to be accommodated and now we see that the accommodation is arrived at by including in the column of revolving fluid an outer region which is approximately represented by the law of the simple vortex with vr constant. It follows that the area of the column of revolving fluid extends far beyond the boundary which was drawn in figs. 4 and 6 to mark the limit of the normal cyclone and includes on the northern side regions of zero velocity and of zero gradient of pressure which in themselves are not at all suggestive of continuous motion in rotation. In order to include the whole rotating mass the boundary circle in the map of fig. 4, chap. x, ought to have extended northward beyond the Scottish mainland and in that case would have taken in also the curved isobar which crosses central France. It would follow that the whole mass of the revolving column occupied the outer margin of the general system

of isobars running from west to east across the map. When we think of the travel of a cyclone we must include those exterior regions which hitherto have seemed to be only part of the environment.

On the other hand, as a suggestion for the boundary of the normal cyclone which formed the central portion of the complete revolving system in the particular case of September 10–11, 1903, the dotted circle of fig. 4, chap. x, is much too large. Its radius is 380 kilometres, whereas, according to the record of wind at Holyhead, the normal cyclone extended only to 125 kilometres from the centre. The limiting circle, as drawn in fig. 4, was taken from the theoretical map of fig. 6 (chap. x), and we may now see the explanation of some of the differences between the two maps. There is a gradually increasing compression of the consecutive isobars in the outer region on the southern side of the theoretical cyclone which is not borne out in the comparison with the actual map. The changes in wind-velocity with distance from the centre stand likewise in need of adjustment. When the theoretical map was drawn it was thought (p. 127) that too large a figure had been taken for the vorticity, but now it appears that for this particular occasion the opposite was the case; the vorticity for the central region was in reality three-and-a-half times the assumed figure; but the radius over which it remained constant was only one-third of that represented. The ring of maximum velocity was much closer to the centre; the immediate environment of the normal cyclone formed part of the revolving system and was a much more extensive and important part of it than was then supposed.

We have noted some curiosities in the relation of wind to the distribution of pressure along the axis of y, which in this case is drawn northward from the centre of the revolving fluid; there will be peculiarities of another kind in other parts of the area, which we have not yet traced and which show that even in the free air, and when the balance of pressure and wind is completely adjusted, the relation of the wind to the distribution of pressure for curved isobars is by no means a direct connexion between the lines of run and separation of the isobars and the direction and velocity of the wind in the same locality. Many more examples must be analysed before the complicated relationships of pressure to wind in the case of revolving fluid can be regarded as established, but if our diagnosis of the conditions of the cyclone of September 10–11, 1903, be correct we may derive some satisfaction for the discomfort which it caused to the International Meteorological Committee, at the Meeting of the British Association at Southport in that year, from the consideration that it was the first to yield a complete analysis of the true internal structure of a travelling cyclonic depression.

We have taken the circular form as typical of the stable condition of rotation but apparently there may be other forms too. The isobars of permanent cyclones are of very diversified shapes and are subject to continual change. The elliptic shape is, however, often persistent and possibly an elliptic form of line of flow is a stable form. A noteworthy example occurred on the map for 7 h on August 29, 1917, in which the major axes of successive

isobars were just twice the minor axes and the distribution of wind showed proportionality to the distance of the isobar from the centre.

The reader will gather from the fragmentary nature of the discussion of the application of equations of revolving fluid to the phenomena of cyclones that the subject is as yet almost unexplored. So far as we have gone, it would appear that the distribution of velocity in the ordinary cyclones of our maps suggests the "simple vortex" with velocity A/r for the outer margin of a cyclone with velocity ζr and the examples of the fast travelling storms of the *Life-History* were not exceptional in that respect. The cyclones of our maps show local distortion of the isobars in the form of secondaries or line-squalls and we have yet to learn how these disturbances become incorporated in the larger general circulation and what is their ultimate effect upon that circulation. A cyclone like that of February 20, 1907, certainly represents the instantaneous motion of a cap of revolving rings about 2600 miles in diameter, or 40° of latitude; and such a mass must have a good deal of stability. At the surface the variations of temperature are very considerable and cause local phenomena of various kinds, but up above we may suppose the distributions of temperature to become as symmetrical as those of pressure or wind.

It has long been supposed that the variations of temperature at the surface are themselves the cause of the original circulation of the cyclone, but it is much more easy to explain convection along the core as the effect of an existing circulation above than *vice versa*, and there are so many examples of convection attended even by copious rainfall which produces no visible circulation that it is difficult to regard convection from the surface as a sufficient cause of our numerous depressions. A useful example may be given from the remarkably wet period of July 27 to August 5, 1917, of the influence produced by rainfall[1]. At the beginning of the period very heavy rainfall occurred in a region of strong winds near Glasgow without any apparent effect on the circulation. But after four or five days of rain about the Straits of Dover, in a persistently quiet environment, a moderate cyclonic depression did appear near that area and it disappeared again after a few days' existence. As the equivalent of the energy set free by the condensation of water-vapour it was very inadequate and we cannot suppose that there was any proportionality between them. Some other conditions than simple convection from the surface are necessary for the development of our cyclonic storms and when they are developed other conditions than convection at the warmest spot provide for their maintenance.

[1] For another example see Daily Weather Reports for January 3–10, 1919.

INDEX

CAMBRIDGE: PRINTED BY J. B. PEACE, M.A., AT THE UNIVERSITY PRESS.